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ON THE CONDITIONS FOR SPARKING AT THE BREAK OF AN INDUCTIVE CIRCUIT.

By J. C. Hubbard.

Introduction.

LORD RAYLEIGH has discussed the relation existing between the energy given to the secondary of an induction coil when the primary current is broken and the velocity of separation of the contacts of the break of the primary.[1] He showed qualitatively, breaking the primary circuit by means of a rifle bullet, that in some cases a greater potential was produced in the secondary than when an ordinary break was used with its *optimum* capacity, and he came to the conclusion that if the break were made with sufficient abruptness, we might do away with the primary condenser altogether.

By optimum capacity is meant that capacity which, when put in parallel with the break of an inductive circuit, is sufficient, and just sufficient, to prevent a spark at the break, when the circuit is broken in a given manner.

We may sum up the chief results of experience in this connection under the following heads:

1. For a given break and primary current there is a given optimum capacity. Walter[2] has shown very clearly by means of figures obtained with a Braun tube that the fall of current in the primary, and therefore the rise of the induced potential, is most

[1] Phil. Mag. (6), 2, p. 581, 1901; Papers, Vol. IV., pp. 564–8.

[2] B. Walter, Ueber die Vorgänge im Inductionsapparat, Wied. Ann., 62, p. 300, Sept., 1897; and 66, p. 623, Nov., 1898.

rapid when the capacity in the primary has been brought to the least possible value without causing the primary break to spark. As the capacity is diminished, the induced potential rises until values are reached so large as to cause an arc to form at the break retarding the rapidity of fall of the current.

2. For larger values of the primary current the optimum capacity is larger. That is, a greater capacity is necessary to prevent a spark.[1]

3. For a given primary current a great increase of abruptness of breaking the circuit diminishes the necessary primary capacity (Lord Rayleigh).

Since the induced potential is proportional to the primary current and inversely to the square root of the capacity it is clear from the foregoing that to get the best possible secondary potentials we must have the largest possible primary currents and the least possible primary capacity, and in order to realize these conditions it is necessary to have the most abrupt possible break.

With a view to determining quantitatively the relation existing between these quantities, J. E. Ives undertook a series of measurements,[2] but he was led to experiment with values of the current so small that he did not succeed in finding the desired relation.

The present paper is an attempt to establish the relation which exists between the rapidity with which the break contacts are pulled apart, or the *velocity of break*, as we shall call it, and the constants of the system together with the primary current, *for a single circuit;* for the results may easily be extended (see supra) to induction coils by the consideration of principles laid down by B. Walter.

THEORETICAL.

Consider the processes going on in an inductive circuit immediately following an interruption of the current. As the lines of magnetic force due to the current disappear, an induced electro-motive force causes a spark at the break, provided the contacts have not become sufficiently separated to prevent it. This spark represents

[1] Walter, loc. cit.; T. Mizuno, On the Function of the Condenser in an Induction Coil, Phil. Mag. (5), 45, pp. 447–454, 1898. K. R. Johnson, On the Theory of the Function of the Condenser in an Induction Coil, Phil. Mag. (5), 49, pp. 216–220, 1900.

[2] PHYS. REV., Vol. XV., No. 1, p. 7, 1902.

a waste of energy and retards the fall of the current. If the velocity of separation of contacts, or the velocity of break, be great enough to prevent a spark, the current will fall to zero with a rate dependent upon its own initial value and the constants of the system.

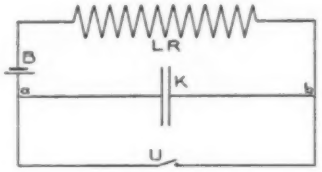


Fig. 1.

Let a circuit be connected as shown in Fig. 1. LR is a coil of inductance L and resistance R. It is connected in series with a battery of electro-motive force E and a break U. Across the break is connected a condenser of capacity K. We wish to find a relation, if any exist, between the velocity of break, the initial current, and the constants of the system when the conditions are just sufficient to prevent sparking at the break.

We have for the equation of the current, on the assumption that there is no spark,

$$L\frac{d^2I}{dt^2} + R\frac{dI}{dt} + \frac{I}{K} = 0,$$

which, with the initial conditions,

$$t = 0, \qquad I = I_0, \qquad Q = 0.$$

where Q is the charge on the condenser, has for its solution

$$I = I_0 \frac{2\sqrt{L}}{\sqrt{4L - R^2K}} e^{-\frac{R}{2L}t} \cos\left\{\sqrt{\frac{1}{KL} - \frac{R^2}{4L}}\cdot t - \tan^{-1}\sqrt{\frac{R^2K}{4L - R^2K}}\right\}.$$

In the experiment which we shall consider K is of the order 10^{-9} farads, L is 1 to 3 henries, and R is 50 to 200 ohms; accordingly, to an approximation having far greater accuracy than we can reach, experimentally,

$$I = I_0 e^{-\frac{R}{2L}\cdot t} \cos\left(\frac{t}{\sqrt{KL}}\right).$$

This equation was used by B. Walter in his experiments with the induction coil.

We have also

$$V = E + L\frac{dI}{dt} - RI,$$

giving

$$V = E + I_0 \sqrt{\frac{L}{K}} \sin\left(\frac{t}{\sqrt{KL}}\right).$$

Since we are dealing with induced electromotive forces of from a few hundred to many thousands of volts we may neglect E, which was usually about two volts. It is clear that the spark, if it takes place at all, will occur some time before the first maximum of potential is reached. Using the above values for the constants of the circuit, the exponential factor $e^{-\frac{R}{2L} \cdot t}$ will have a value of about .99 at the end of the first quarter period. We therefore have, to an approximation of about one per cent.,

$$V = I_0 \sqrt{\frac{L}{K}} \sin\left(\frac{t}{\sqrt{KL}}\right).$$

We consider now the action of this potential in producing a spark at the break, assuming that the contact surfaces are portions of spheres of large radius in comparison with sparking distances.

In the first place, we have a rapidly varying potential. Does a varying potential follow the law which connects a constant potential with its spark length? There has been much controversy on this point. Jaumann[1] maintained that the spark length for a varying potential does not depend upon the value of the potential at the instant of sparking but upon the value of VdV/dt. As J. J. Thomson has pointed out,[2] his experiments may be brought into accord with the law for constant potentials by considering the electric waves set up in the system. Some work of Warburg[3] seems to settle the question, for he shows that within limits potential changes are without influence on the sparking constants, provided the gas is ionized.

Further, there is the question of the lag effect on the spark length. Jaumann[4] showed that there is an interval between the time of application of the potential and the time of sparking, during which some

[1] Wied. Ann., 55; p. 656, 1895.
[2] Conduction of Electricity through Gases, p. 351.
[3] Wied. Ann., 62, p. 385, 1897; Ann. d. Phys., 5, p. 818, 1901.
[4] Loc. cit.

process goes on in the gas which changes it into a conductor. J. J. Thomson applied without causing a spark a potential several times the sparking value to a spark gap in carefully dried air. The complete determination of the lag effect would require more knowledge than we have at present of the growth of ionization produced by an electric field. According to J. J. Thomson's theory of the spark discharge[1] when a sufficiently great difference of potential is applied to a spark gap, ions initially present are driven with great enough velocity to produce fresh ions by collision with molecules of the gas. These new ions produce ions in their turn and the ionization and accordingly the conductivity of the gas will increase in geometric progression until the spark takes place. The duration of the lag will depend on the number of ions initially present. The lag is much diminished by any ionizing agency, such as ultra violet light, or the spark itself.

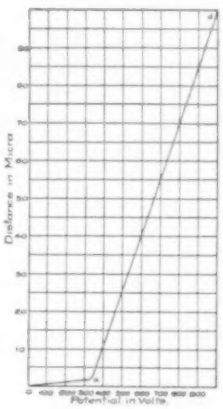


Fig. 2.

Earhart[2] has obtained values of the sparking potential for small distances between spherical surfaces in air. The relation is shown in Fig. 2 taken from Earhart. For distances from zero to .003 mm. the potential is proportioned to the distance (*oa*, Fig. 2). Here the spark potential, $V_s = gx$, where g is about 1.2×10^6 volts per cm. The curve changes abruptly between $x = .003$ mm. and $x = .01$ mm. after which it follows the law $V_s = ax + b$, where a is about 70,000 volts per cm. and b is less than the minimum spark potential, 351 volts. In this paper we shall deal only with the branch of the curve *ad*, Fig. 2, where $V_s = ax + b$. We shall assume the equation to hold regardless of variations of potential, and endeavor to find by means of deviations from it some indication of the magnitude of the lag effect in the cases under consideration.

We have then, as the limiting conditions for sparking,

$$V_s = V$$

and

[1] Phil. Mag. (5), 50, p. 278, 1900.

[2] Earhart, Phil. Mag., VI., 1, p. 147, 1901.

$$\frac{dV_s}{dt} = \frac{dV}{dt},$$

also, $x = vt$, where v is the velocity of break. Therefore,

$$avt + b = I\sqrt{\frac{L}{K}} \sin \frac{t}{\sqrt{KL}},$$

and

$$av = \frac{I}{K} \cos \frac{t}{\sqrt{KL}}$$

where I is the initial current. Eliminating t,

$$av\sqrt{KL} \cos^{-1}\frac{avK}{I} + b = I\sqrt{\frac{L}{K}}\sqrt{1 - \left(\frac{avK}{I}\right)^2}.$$

In the present experiments $avK/I = Z$ was seldom as much as .4, that being one of the largest values recorded.

We have

$$\cos^{-1} Z = \tfrac{1}{2}\pi - Z - \frac{Z^3}{6} \cdots,$$

$$(1 - Z^2)^{\frac{1}{2}} = 1 - \tfrac{1}{2}Z^2 - \tfrac{1}{8}Z^4 \cdots.$$

Neglect of higher powers of Z than the second will cause an error under one per cent. Accordingly

$$av\sqrt{KL}\left(\frac{\pi}{2} - \frac{avK}{I}\right) + b = I\sqrt{\frac{L}{K}}\left[1 - \frac{1}{2}\left(\frac{avK}{I}\right)^2\right],$$

or

$$\frac{\pi}{2}\cdot\frac{avK}{I} - \frac{1}{2}\left(\frac{avK}{I}\right)^2 + \frac{b}{I}\sqrt{\frac{K}{L}} = 1.$$

As a first approximation

$$\frac{\pi}{2}\cdot\frac{avK}{I} + \frac{b}{I}\sqrt{\frac{K}{L}} = 1.$$

Solving for avK/I and substituting its square in the first equation,

$$\frac{\pi}{2}\left(\frac{avK}{I}\right) = 1 - \frac{b}{I}\sqrt{\frac{K}{L}} + \frac{2}{\pi^2}\left(1 - \frac{b}{I}\sqrt{\frac{K}{L}}\right)^2$$

whence

$$v = \frac{2I}{\pi aK}\left(1 + \frac{2}{\pi^2}\right) - \frac{2b}{\pi a\sqrt{KL}}\left(1 + \frac{4}{\pi^2}\right) + \frac{4b^2}{\pi^3 alL}.$$

or

$$(1) \qquad v = .7656\frac{I}{aK} - .8946\frac{b}{a\sqrt{KL}} + .1290\frac{b^2}{aLI},$$

the equation connecting the velocity of break with the constants of the system and the initial current, which we shall use in connection with the experiments.

We may write the first approximate equation thus:

$$v = \frac{2}{a\pi\sqrt{KL}}\left(I\sqrt{\frac{L}{K}} - b\right),$$

showing that the velocity of break necessary to prevent a spark is roughly proportional to the requency and to the excess of the maximum potential over the minimum spark potential.

Experimental Arrangements.

The apparatus was usually arranged in the following manner: An inductance, LR, storage cell, B, interrupter U, and box resistance R_b, were connected in series as shown in Fig. 3. A condenser K of variable capacity is joined in parallel with the box resistance and break. The purpose of this resistance is to permit variations of current without affecting thedamping factor. A quadrant electrometer is connected idiostatically with the terminals of the condenser. The inductance used has been described by J. E. Ives.[1] It was wound in a series of six coils in a large flat wooden spool. The total inductance is 3.01 henries. About two miles of wire was used, having a resistance of 62.18 ohms. A bridge circuit, not shown, was arranged for the purpose of measuring the resistance of various parts of the circuit, or of all together, at will. The terminal of the condenser next the battery was kept to earth at E. The capacity of the system is that of K with one of its plates to earth, together

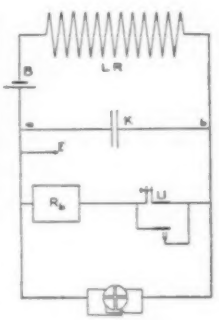


Fig. 3.

[1] J. E. Ives, Phys. Rev., XIV., p. 298, 1902.

with the capacity of its leads, of the break with its contacts apart and its leads, and the unknown, and in these experiments unmeasured capacity of the coil due to the oscillations of the potential. The capacity of the coil is probably negligibly small in comparison with the external capacity, the more so, as Drude[1] has shown that the capacity of a coil vibrating in its natural period is lessened with the addition of end capacity.

For a break was used one of the contact levers of Dr. Webster's drop chronograph.[2] The lever is so adjusted that when the break is closed the end which is struck by the falling weight is on a level with the pivot, ensuring that the velocity of the end when struck, in the direction perpendicular to the radius is equal to the velocity of the falling weight. The velocity of break is then equal to $r\sqrt{2gh}$ where $\sqrt{2gh}$ is the velocity of the projectile and r is the ratio of the length of the arm carrying the contact to the length of the arm struck having the value .479.

MEASUREMENT OF CAPACITY.

The most trouble encountered in the experiments was in connection with the measurements of capacity. The capacity of the system is of the order 10^3 cm. which is difficult to measure electrostatically with a great degree of accuracy. The method of measurement adopted was suggested by Dr. Webster. K_2 (Fig. 4) is a standard condenser. K_1 is the system the capacity of which is to be measured. The part of the system which was connected to earth in the oscillation experiments is the upper part of K_1, which together with the upper side of K_2 may be connected to earth at E_1, at a, but always connected with the electrometer. A steady current flows from a large number of storage cells B through R_1 and R_2 between which is an earth connection, E_3. A commutator connects 2 with

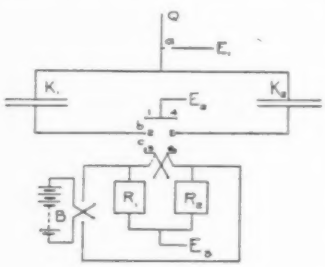


Fig. 4.

[1] Drude, Ann. d. Phys., 314, p. 293, 1902.

[2] A. G. Webster, An Experimental Determination of the Period of Electrical Oscillations, PHYS. REV., VI., p. 300, 1898.

3, and 5 with 6, at c. The lower plates of K_1 and K_2 are thus at potentials proportional to R_1 and R_2. Equal and opposite charges are induced on the opposite branches of the system. If, now, the earth connection at a be broken and the commutator rocked so as to connect 1 with 2 and 4 with 5 at b, the charges on the lower plates run to earth at E_2 and, if the charges on the upper plates be equal, no deflection in the electrometer will result. Then will

$$\frac{K_1}{K_2} = \frac{V_2}{V_1} = \frac{R_2}{R_1}.$$

All possible precautions were taken to insure perfect insulation. All connections were of copper, as it was found that the use of mercury connections introduced serious error, as from heating by a nearby lamp, or mechanical agitation, producing thermo-electric effects. It was found necessary to cement all binding posts to glass with sealing wax and to have them sufficiently removed from other conductors to make their capacity negligible. The switches and commutator were made of small metallic parts embedded in beeswax or paraffine. On dry days it was found possible and easy to measure capacities of 10^3 cm. to 1 cm. The resistances were reversed, the direction of current was reversed and various points of connection were shifted without causing changes greater than those observed when the conditions were kept the same. On damp days it was found possible to measure capacities only after all connections had been carefully gone over with a jet of air which had been passed through a quantity of concentrated H_2SO_4. To test the method different lengths of parallel wires were connected to K_1 and the increase in capacity thus produced agreed with that calculated to 1 cm.

The variable capacity consisted of two plates, the upper one being in the form of a small plate with a guard ring which were connected together so as to act as one plate with a diameter of 18 inches. This plate was supported at the edge upon ebonite pillars which rested upon a rigid iron frame work. Beneath this was the other plate, 16.5 inches in diameter, supported by a large screw which, by means of a nut at the base of the instrument, could be raised or lowered without rotation. The screw threads were 20 to the inch.

One revolution of the nut from the position which gave contact (not taken as the zero) gave 820 cm. capacity.

The standard condenser has been fully described by Dr. Webster.[1] The lower plate was supported upon three rigid, ebonite tipped legs. On top of the plate, directly over its points of support, were placed three small glass cylinders, of accurately known length, and upon these was set the top plate. The condenser was kept throughout at some distance from the other apparatus, and at about 5.5 feet distance from the floor and eight feet from the nearest wall. On the approach of a person as closely as possible without touching, the condenser showed a change of capacity of about one fourth per cent.

It is of course necessary to keep the form of all circuits, connections, and apparatus the same during measurements of capacity as they are kept when experiments are made in which use is to be made of capacity measurements. There must be no electrometer deflection when the earth connection at a is broken, whether K_1 or K_2 is connected in alone, or both together. This was the test for leakage in the capacities. We are thus able to measure with considerable accuracy all the capacity effective in the oscillation experiments with the exception of that due to the coil. The variable condenser was calibrated. On the addition of the electrometer with its leads the increase of capacity was found to be 13 cm. On the addition of the leads to the break the increase was 12 cm. In the oscillation experiments, accordingly, the reading of the condenser screw was taken, the capacity found from the calibration curve, and 25 cm. added. This was occasionally verified by the measurement of all together.[2]

An example is here given of the magnitude of the changes produced in the capacity measurements when the currents and resis-

[1] Loc. cit.

[2] A slight modification of the above method enables us to measure the capacity of a small body with reference to all surrounding bodies taken together. Put a at E_1 permanently to earth, and make the electrometer connection at E_2. Then when the commutator is rocked after disconnecting E_2 from earth, if the charge on the lower parts of K_1 and K_2 are equal no deflection will result. A spherical acoustical resonator of 23 cm. diameter, having a slight projection on one side, was supported at a distance of three feet from all other conductors. It was put in connection with the lower side of K_1. The observed increase in capacity was $12 \pm .25$ cm.

tances were reversed. The nut of the variable capacity was turned .588 revolution from zero position. Twenty storage cells were used in the battery.

	R_1	R_2	R_2/R_1
	640.5	1,000.0	1.5613
Current reversed.	639.5	1,000.0	1.5637
	1,000.0	1,565.5	1.5655
Current reversed.	1,000.0	1,565.5	1.5655

$$K_2 = 826.5 \text{ cm.} \quad K_1 = \frac{R_2}{R_1} K_2 = 1{,}293 \text{ cm.}$$

The following experiments do not call for this degree of accuracy and it might seem that the large amount of time spent upon the capacity measurements was lost, but in such work a very small amount of leakage causes indefinitely large errors and care had to be taken that everything of the kind should be eliminated.

A proof of the relative correctness of the calibration of the variable condenser is found in the fact that the calibration curve is a very smooth one and follows the formula

$$K = \frac{c}{d + e}$$

over a long range, c being a constant, d is the reading of the screw from zero position and e is a small correction to be added to d for the reason that the plates, owing to imperfect surfaces and lack of parallelism did not everywhere touch when $d = 0$. Part of the calibration is here given to show the agreement between measured values of K and those calculated from the above formula, using $c = 863.5$ and $e = .080$.

d	K Meas.	K Calc.	Diff.
.610	1,252	1,252	0
.660	1,170	1,167	−3
.720	1,079	1,079	0
.800	980	982	+2
.890	891	890	−1
1.000	799	799	0

The formula takes no account of the thickness of the plates or of the metallic frame work.

METHOD OF TAKING READINGS.

In finding the values of velocity of break just sufficient to prevent sparking, the apparatus was connected as in Fig. 3. When the circuit is broken at U, there will result oscillations of potential in the condenser and at the break which are measured by the electrometer, connected as shown. The drop, in continuing its fall, after breaking the circuit closes it again by means of another lever, indicated in the figure, after the potential oscillations have died away, but in so short a time as to eliminate the effect of the battery e.m.f. on the readings of the electrometer. The latter are then proportional to the time integral of the square of the induced potential. In all the following experiments except the first, the constants of the circuit were kept the same in each set while the current was varied. The electrometer readings, provided there is no spark, will then be proportional to the square of the initial current.

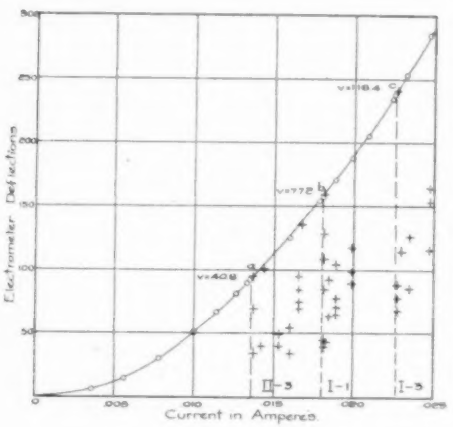


Fig. 5.

Imagine the projectile of the drop chronograph hung at a distance h above the contact lever of the break. Each interruption of the current caused by the fall will be made with the same velocity of break. Make R_b very large so as to cause a small current to flow through the inductance when the circuit is closed. If the drop is allowed to fall, a definite deflection will result, and will be the same for every fall if there is no spark. Let smaller values of R_b be chosen, so as to increase the initial current. There will be continuously larger electrometer deflections, and the curve plotted from initial currents and electrometer readings (see Fig. 5) will be parabolic in form as long as no sparking takes place. As

we go on increasing the initial current we at length come to a point (at *a*, for instance) beyond which the electrometer readings fall short of the curve and differ among themselves by several hundred per cent. An examination of the break reveals the fact that a small spark takes place at each interruption. The region of this discontinuity was carefully explored. The limiting value of the current is taken as the largest value of the current which will give five successive electrometer readings on the curve when the circuit is broken. This rule was adopted both to eliminate personal judgment from the results, and because it was often found possible by careful polishing of the break contacts to get sparks after two or three readings had been taken which lay on the curve, showing that the critical value had not been reached. Points somewhat separated were chosen for Fig. 5, so as to show the regions explored without overlapping.

The following table for the exploration of the neighborhood of a point of discontinuity (*c*, Fig. 5) will show in what manner the limiting values of the current corresponding to velocities of break were chosen. The last two rows of figures give the number of readings on and off the curve, respectively,

$K = 585$ *cm.* $L = 3.01$ *henry.* $R = 62.2$ *ohms.* $h = 30.02$ *cm.* $v = 116.4$ *cm./sec.* $E = 2.029$ *volts.*

R_b	10	20	24	26	27	27.5	28
I.	.02816	.02471	.02354	.02302	.02278	.02263	.02253
No spark.	0	0	0	0	3	5	5
Spark.	2	3	2	1	3	0	0

giving $I = .02272$ ampere, corresponding to $R_b 27.3 \pm 2$ (see supra, series 1–3). The velocities corresponding to *a* and *b* of the curve were respectively 40.8 (series II. — 3), and 77.2 cm./sec. (series I. — 1).

Values obtained in this way were supplemented by direct observations at the spark gap and were always verified. It was thus possible to determine the limiting values of the current to one per cent. The electrodes of the break were carefully polished with a silk cloth after each reading. The electromotive force of the battery was measured whenever a limiting value of the current had been

reached, and the resistance of the entire circuit including the break, closed, was found at the same time. Occasional verifications of the magnitude of the capacity were made. The switch for releasing the drop and the telescope for reading the electrometer, as well as the commutator for the purpose of bringing its needle to rest were placed on a table at a distance of several feet from the oscillation circuit.

PRELIMINARY EXPERIMENT.

Connection Between Optimum Capacity and Velocity of Break.

An experiment was first undertaken to ascertain suitable values of the quantities involved for the purpose of later work. The system was set up as in Fig. 3 omitting R_b. The variable capacity was made large and gradually diminished until the electrometer readings on breaking the circuit at a definite rate ceased to be a continuous function of the capacity. The region of discontinuity was explored, much as described in the previous section. The optimum capacity was chosen as the least capacity which gave five successive readings which lay on the capacity-deflection curve. The experiment was undertaken before much progress had been made with the measurements of capacity and the accuracy of the latter is probably not much greater than five per cent. The following table gives the values of the optimum capacity with the corresponding velocities of break:

$L = 3.01$ *henry*. $R = 63$ *ohms*. $R_b = 0$. $E = 2.02$ *volts*. $v = r\sqrt{2gh} = 21.2\sqrt{h}$.

h cm.	v cm./sec.	K cm.
69.1	176.2	670
15.5	83.5	1,160
4.8	46.4	1,780
1.11	22.3	4,170

The curve got by plotting K and v is almost hyperbolic in form (Fig. 6) showing that for a given current the optimum capacity is nearly inversely as the velocity of break.

If we put for equation (1) when K is in farads

$$v = \frac{A}{K} - \frac{B}{\sqrt{K}} + C$$

we get from the above data,

$$A = 238{,}000.$$
$$B = 6{,}270.$$
$$C = 66.4.$$

Where

$$A = .7656\,\frac{I}{a}; \quad B = .8946\,\frac{b}{a\sqrt{KL}}; \quad C = .1290\,\frac{b^2}{aL}.$$

From A,

$$a = 92{,}800 \text{ volts/cm.}$$

from B and a,

$$b = 1{,}190 \text{ volts.}$$

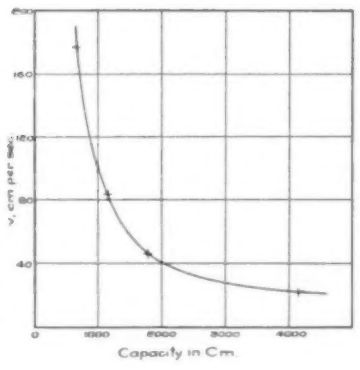


Fig. 6.

The value of a as obtained by Earhart is, as we have seen, about 70,000 volts/cm., while that of b is 351 volts. The agreement for a is perhaps as good as could be expected in view of the small number of readings, the errors of capacity measurements, and the neglect of the lag effect.

Experimental Results.

The following results were obtained by the method described in the last article but one. The capacities were measured with an accuracy of from one tenth to one third per cent., not including the capacity of the coil. The limiting values of the current correspond-

Series I.

$K = 585$ *cm.* $L = 3.01$ *henry.* $R = 62.2$ *ohm.*

	R_b	$\pm$	E	I	h	v
1	50.0	2.0	2.032	.01810	13.29	77.2
2	37.7	.2	2.032	.02032	22.08	99.6
3	27.3	.2	2.029	.02272	30.02	116.4
4	19.2	.2	2.029	.02496	40.89	135.4
5	18.5	.5	2.030	.02517	40.89	135.4
6	10.2	.2	2.030	.02806	56.01	158.8
7	75.0	5.0	2.030	.01482	6.59	54.5

ing to the velocity of break were measured within one per cent. except where specified. The inductance and velocity of break are known to a much greater degree of accuracy. All quantities are in the usual units except the capacity, kept in cm. for convenience.

The total resistance, from which I is calculated, is $R_b + R$ plus the small resistance of the leads and break, which was found to be negligible in the above results. The accuracy of the results 2–6

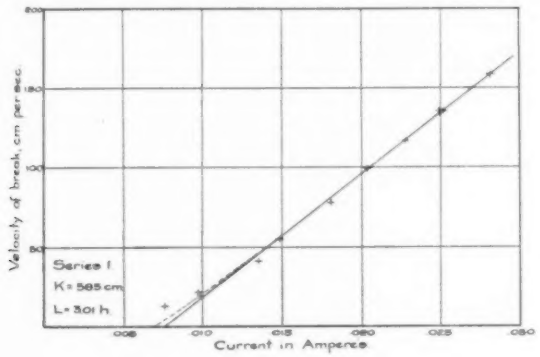


Fig. 7.

is thus seen to be well within one per cent. The currents and velocities of break are plotted as abscissæ and ordinates respectively, being all but the lowest three points of Fig. 7 which belong to the next series. It will be seen that the relation is approximately a linear one.

We may write equation (1) thus:

$$v = AI - B + \frac{C}{I}, \tag{1}$$

where C has the value $.1290b^2/aL$. For the moment let us assume for a and b the values obtained by Earhart. Taking representative value of the current, .025 am. say, the third term on the right of (1) has the value 3 cm./sec. It will differ from this by only one or two for any other value of the current in the range. The velocities run from 50 to 160 cm./sec. Accordingly the C term in the above equation is in the nature of a small correction to the equation.

$$v = AI - B, \tag{2}$$

the asymptote of (1). Selecting the values 2–6 in the above results as being much more accurately determined than the other two, let the straight line determined by them be

$$v = A'I - B'. \tag{3}$$

Then

$$A = 7{,}751\,; \quad B' = 58.8.$$

We have, from values of A, B and C, already given,

$$C = \frac{.1290 \times .7656}{(.8946)^2}\,\frac{B^2}{A}$$

Substituting the approximate values A' and B',

$$C = .055$$

The slope of the curve (3) is A' and is sensibly equal to the slope of the curve (1), especially for larger currents. We have then, for the current $I_1 = .025$ am. say,

$$A = A' + C/I_1^2 = 7{,}839,$$

which will differ by one per cent. or less if we choose any other current value in 2–6 as the basis of calculation. Similarly,

$$B = AI_1 - v_1 + C/I_1 = 63.1$$

where v_1 is taken from the curve (3) to correspond to I_1.

The curve $v = 7{,}839I - 63.1 + .055/I$ is shown deviating from the straight line to the left in Fig. 7. What we have done in getting the constants is equivalent to assuming that the curve (1) is tangent to (3) at the point (I_1, v_1). The justification lies in the fact that the curve (1) satisfies the points in the range of observation 2–6 as well as the line (3).

Since

$$A = 7{,}839 = \frac{.7656}{aK}, \quad a = 150{,}000 \text{ volts/cm.},$$

and since

$$B = 63.1 = \frac{.8496b}{a\sqrt{KL}}, \quad b = 474 \text{ volts.}$$

The constant a comes out about double the value found by Earhart and b comes out one third greater. We shall see that this increase is clearly explained by the lag effect.

In showing the order of magnitude of the C term in (1) we used Earhart's values for a and b. If we use our own values it will come even smaller.

It is clear from this experiment that the lag effect is independent of the initial current, that is, of the magnitude of the induced potential, when the constants of the circuit are kept the same, since we get an approximately linear relation for v and I in agreement with the theory. Jaumann's hypothesis that the spark lengths for rapidly varying potential depend upon a definite value of VdV/dt is thus not borne out.

An attempt was next made to get values of v for very small initial currents. We should expect, when we approach induced potentials of the order of the minimum spark potential, to get a change in the spark constants, as in Earhart's results (Fig. 2), where, for $V<351$ volts, we have $V = gx$, where g is about 1.2×10^6 volts /cm. For this branch of the curve the relation between the velocity of break, current, and constants of the circuit may be found by putting $b = o$ and $a = g$ in eq. (I). Then we have $v = .7656\ I/gK$. Accordingly, for smaller and smaller currents, we should at length come to an abrupt deviation from the straight line intersecting the I axis as in Fig. 7 and the points would rapidly approach the origin.

The readings for such small currents were very difficult to obtain owing both to the small readings of the electrometer, and to the minuteness of the spark, which rendered it difficult to determine whether the spark had or had not taken place. The following values were obtained:

SERIES II.

Constants as in Series I.

	R_h	±	E	I	h	v
1	205	5	2.028	.00760	.390	13.2
2	146	1	2.025	.00973	1.05	21.7
3	87	2	2.025	.01357	3.70	40.8

giving the lowest three points in Fig. 7. We have perhaps an indi-

cation of the change in the first two points. For the first the maximum induced potential is 517 volts.

Experiments were next tried to find whether the spark constants, not affected by changes of current, would be affected by change of capacity.

SERIES III.

$K = 789$ *cm.* $L = 3.01$ *henries.* $R = 62.2$ *ohms.*

	Total R	E	I	k	v
1	101.0	2.043	.02020	9.65	65.8
2	73.0	"	.02801	30.45	116.9
3	71.5	"	.02856	31.67	119.3
4	71.2	"	.02870	34.30	124.1
5	69.0	"	.02964	38.56	131.6
6	81.2	"	.02516	21.07	97.2
7	172.6	4.093	.02372	17.10	87.6
8	154.8	4.088	.02642	24.99	106.0
9	228.8	4.085	.01786	6.41	53.7
10	197.8	"	.02066	11.79	72.8
11	165.8	"	.02464	18.49	91.2

The straight line given by (I, v) is plotted in Fig. 8, No. III.

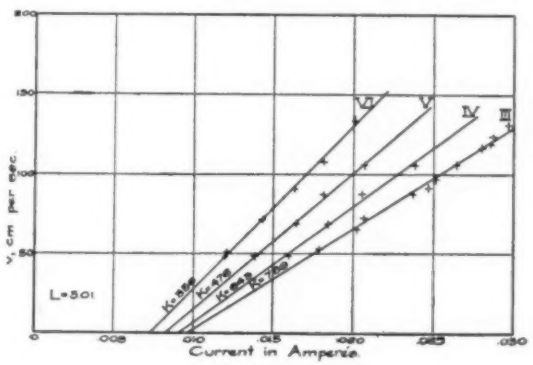


Fig. 8.

Using the same method as before for obtaining A, B, and C, we obtain

$$A = 6{,}510; \quad B = 67.9; \quad C = .074.$$

Whence

$$a = 134{,}100 \text{ volts/cm.}$$

$$b = 523 \text{ volts.}$$

SERIES IV. (Fig. 9, *II.*)

K = 643 *cm. L and R as in III.*

	Total *R*	*E*	*I*	*h*	*v*
1	256.3	4.088	.01594	5.16	48.1
2	222.6	4.093	.01838	10.44	68.5
3	199.6	4.093	.02052	17.10	87.6
4	171.8	4.088	.02380	24.99	106.0

From which

$$A = 7{,}578; \quad B = 75.3; \quad C = .077.$$

and

$$a = 141{,}400 \text{ volts/cm.}$$

$$b = 552 \text{ volts.}$$

SERIES V. (Fig. 9, *III.*)

K = 476 *cm. L and R as before.*

	Total *R*	*E*	*I*	*h*	*v*
1	294.7	4.088	.01386	5.16	48.1
2	249.6	4.093	.01641	10.44	68.5
3	225.8	4.088	.01812	17.10	87.6
4	197.8	4.088	.02066	24.99	106.0

giving

$$A = 8{,}748; \quad B = 77.7; \quad C = 0.71.$$

$$a = 165{,}500 \text{ volts/cm.}$$

$$b = 573 \text{ volts.}$$

SERIES VI. (Fig. 9, *IV.*)

K = 356 *cm. R and L as before.*

	Total *R*	*E*	*I*	*h*	*v*
1	338.7	4.088	.01206	5.16	48.1
2	336.8	4.073	.01210	5.96	51.7
3	284.8	4.073	.01431	11.40	71.6
4	249.8	4.073	.01634	18.31	90.7
5	224.8	4.070	.01811	25.85	107.8
6	202.8	4.070	.02006	39.63	133.4

giving

$$A = 10{,}390; \quad B = 81.5; \quad C = .066.$$

and

$$a = 186{,}200 \text{ volts/cm.}$$

$$b = 585 \text{ volts.}$$

From Fig. 9 it is seen that for diminishing capacity, the curves (I, v) progress toward the origin and increase in slope, agreeing with equation (1). But the increase in slope is not so great as would be expected, for the quantity a steadily increases, as we may see from the following summary of series I., III.–VI.

Series.	K cm.	a	b
III.	789	134,100	523
IV.	643	141,400	552
I.	585	150,000	474
V.	476	165,500	573
VI.	356	186,200	585

The quantity a and the quantity b, with one exception, are seen to increase with diminishing capacity. Without, for the present, considering the cause, we pass to experiments in which the capacity was kept constant while the inductance was changed.

Series VII.

$K = 789$ *cm.* $L = 1.97$ *henries.* $R = 50.9$ *ohms.*

	Total R	E	I	h	v
1	188.9	4.073	.02159	6.62	54.6
2	156.9	"	.02600	13.41	77.7
3	128.9	"	.03164	27.42	111.0
4	186.9	4.079	.02184	6.12	52.4
5	159.0	"	.02570	12.95	76.3
6	136.9	"	.02982	22.97	101.6

From these

$$A = 5{,}967\,; \quad B = 81.1\,; \quad C = .113,$$

giving

$$a = 146{,}300 \text{ volts/cm.}$$

$$b = 551 \text{ volts.}$$

SERIES VIII.

$K = 789$ *cm.* $L = 1.32$ *henries.* $R = 38.9$ *ohms.*

	Total R	E	I	h	v
1	150	4.073	.02721	6.69	54.8
2	119	"	.03430	16.50	86.2
3	105	"	.03888	26.60	109.2
4	128	"	.03188	11.01	70.4

From which

$$A = 4{,}845\,; \quad B = 84.6\,; \quad C = .152,$$

giving

$$a = 180{,}200 \text{ volt/cm.}$$
$$b = 580 \text{ volts.}$$

These two series with III., in which the capacity was the same, are shown in Fig. 9. We notice progression away from the origin

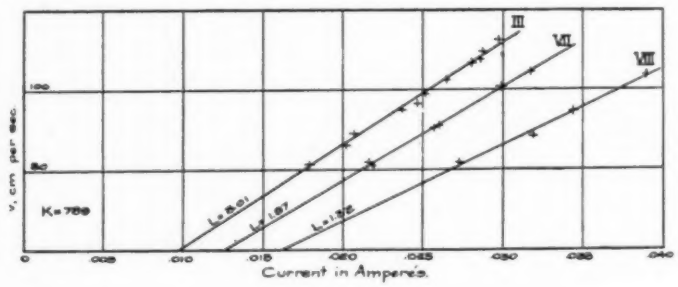


Fig. 9.

with diminishing inductance, as we should expect from the theory. But, instead of the slope being practically constant, it diminishes with the inductance. We show the results together for change of inductance.

$K = 789$ *cm.*

Series.	L	a	b
III.	3.01	134,100	523
VII.	1.97	146,300	551
VIII.	1.32	180,200	580

We notice increase of both quantities a and b with diminishing inductance, as with diminishing capacity.

We close with one more series, in which the period is less than that in any previous set.

SERIES IX.

$K = 356$ *cm.* $L = 1.97$ *henries.* $R = 50.9$ *ohms.*

	Total R	E	I	h	v
1	216	4.079	.01882	13.50	77.9
2	189	"	.02161	23.98	103.8
3	171	"	.02390	34.98	125.3
4	203	"	.02014	17.91	89.7

giving

$$A = 9{,}550\,; \quad B = 108.3\,; \quad C = .125.$$

$$a = 202{,}700 \text{ volt/cm.}$$

$$b = 685 \text{ volt.}$$

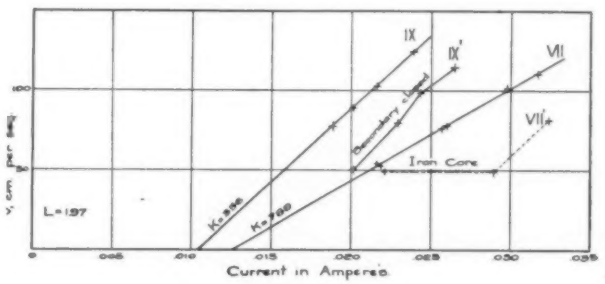


Fig. 10.

These points, with which those of series VII., are plotted in Fig. 10 for purposes of comparison.

DISCUSSION OF RESULTS.

It might appear at first sight that the unknown capacity of the coil would explain the variation of the quantities a and b from the accepted values. If we assume $a = 70{,}000$ volt/cm. and calculate from the constants A and the capacities made necessary by the assumption, we shall find that they are all the way from two to three times the capacity which was measured. Moreover, the excess of apparent capacity over the measured capacity in series I.–VI. will be found to diminish as the latter diminishes, which is contrary to what we should expect if we were to attribute the

apparent excess to the coil. Again, in the preliminary experiment, no perceptible difference was noticed, whether the battery side of the circuit was or was not earthed. We may therefore assume that the whole of the coil is at practically a node of potential and that its capacity is negligible.

We may very simply explain the results on the basis of any theory which takes into account the fact that an appreciable time must elapse after the application of a potential to a spark gap before a spark can take place, some process going on which converts the gas between the electrodes from an insulator into a conductor.

Such a theory has been developed by J. J. Thomson.[1] He obtains for the equation for the production of ions by an electric field between two parallel plates

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nu) = \frac{nu}{\lambda}\{f(Xe\lambda) - \gamma\}.$$

where n is the number of ions per cubic centimeter, X is the electric force u, λ, e, and $Xe\lambda$ are respectively the average velocity, mean free path, charge and mean kinetic energy of an ion. γ is a constant.

The complete solution of the above equation for n will be an exponential function of the time. The density of ionization and, accordingly, the conductivity of the gas will increase in geometric progression until they reach the required value for sparking, when $\partial n/\partial t$ becomes zero. With the electric force and spark length kept constant, the condition that the gas has reached the steady state, gives for the spark potential, according to J. J. Thomson,

$$V = ad + b,$$

for spark lengths, d, greater than the critical spark length, a and b being constants.

For the purposes of this paper we require the solution of the differential equation for the following condition: Given a spark length d, what will be the potential necessary to produce a spark at the time t? It seems probable that it will be of the form

[1] J. J. Thomson, Phil. Mag. (5), 50, p. 278, 1900. See also Conduction of Electricity through Gases, pp. 231, 341, 379.

$$V = (a + m)d + b + n.$$

When m and n are such functions of the time that when t approaches zero they become indefinitely large and, conversely, when t becomes large they will become zero and V will have its normal value. m and n are thus measures of the lag effect.

Although we have V and x, the spark length, as functions of the time, we may show that the conditions of our experiments are such that the electric force will be only dependent upon the time to the second order. This will be evident without proof if we consider that the conditions are such that the limiting condition for sparking requires that the induced potential and the spark gap increase at roughly the same rate. The problem is thus reduced approximately to the one stated above.

We may show also that the electric force, under the experimental conditions, is such as to be independent of the current for larger values of the latter, for we found by experiment that $v = AI - B + C/I$. The spark length is proportional to v. The electric force, then, will contain I only in the denominators of the small terms. This gives us justification for calculating a and b from A and B of the previous series, for that calculation involves the assumption that the lag effect is constant for a given period regardless of the initial current.

The constants m and n will depend upon the time at which the spark takes place. This time, as given in the first part of this paper is

$$t = \surd KL \cos^{-1}\left(\frac{a'vK}{I}\right) = \surd KL\left(\frac{\pi}{2} - \frac{a'vK}{I} \cdots\right)$$

where $a' = a + m$. We have seen that $a'vK/I$ is small in the limits of our experiment, being from .2 to .4. We may therefore consider the time required to be roughly proportional to the period, and for purposes of illustration we may take m and n as such functions of the period that they will vanish when the latter is infinitely large, and become themselves infinite when the period approaches zero. This will be true in general for m at least, whatever theory of ionization we adopt. The simplest functions which will satisfy the relation will be

$$a' = a + m = a\,(1 + \alpha/\sqrt{KL})$$
$$b' = b + m = b\,(1 + \beta/\sqrt{KL})$$

where a' and b' are the quantities previously called a and b in the tables, and α and β may be considered as time constants of the gas.

The following results from series I.–IX., with the preliminary experiment P, show the dependence of a' and b' upon the period. Three significant figures are kept.

Series.	$\sqrt{KL} \times 10^5$.	$t \times 10^5$.	a'	b'
IX.	2.79	3.30	203,000	685
VIII.	3.40	3.88	180,000	580
VI.	3.45	3.86	186,000	585
V.	3.99	4.52	165,000	573
VII.	4.16	4.82	146,000	551
I.	4.42	4.59	150,000	474
IV.	4.64	5.43	141,000	552
III.	5.14	5.76	134,000	523
P.	8.05	10.14	93,000	

The time t is calculated from the formula just given, using the mean value of $a'vk/I$ in series I.–IX., and in the preliminary ex-

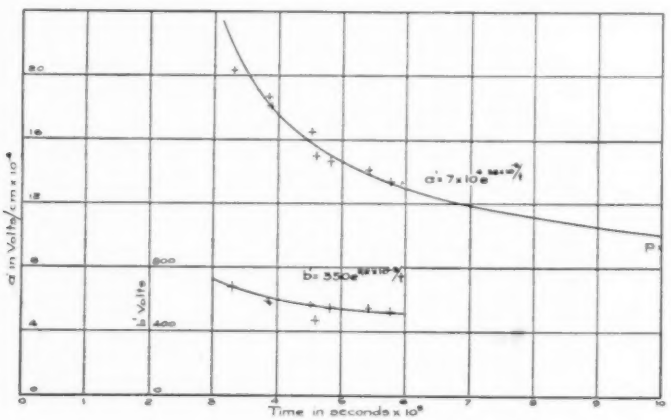


Fig. 11.

periment using the mean value of K. The plots of a' and b' with the times as abscissæ are shown in Fig. 11.

The function $a' = a(1 + \alpha/t)$ seems to satisfy the results of series I.–IX. as well as any function which could be chosen. But it

comes very wide of the point P, obtained from the preliminary experiment, which, while given no weight in the calculations, is supposed to be determined within five per cent. The spark constant a, calculated from the results will be about 5×10^4 volt/cm. in place of Earhart's value of about 7×10^4. Though our data are not sufficiently extensive to enable us to tell definitely, it would seem that the function m must be some function of the time which diminishes more rapidly as the time increases than does $a\alpha/t$.

If we put $a' = ae^{\alpha/t}$, and assume $a = 70{,}000$ volts/cm., the mean value of α from the nine series is 3.6×10^{-5} sec. The curve thus obtained is the upper one in Fig. 11. It gives a value within 8 per cent. of the a' given by the preliminary experiment.

The curve obtained through the observations b' is obtained by putting $b = 350$ volts and $\beta = 2.2 \times 10^{-5}$ sec. in the equation $b' = be^{\beta/t}$.

Addenda.

Though the aim of the work here described was to study the conditions for sparking at the break of the primary circuit, some experiments were made with the view of finding what the condition would be for the primary of the induction coil with iron core and with closed secondary. The presence of an open secondary was found to be without perceptible effect on the necessary velocity of break for a primary circuit.

Secondary Closed. — In Series IX. the inductance used consisted of the first five parts of the coil described by Dr. J. E. Ives,[1] the sixth part being disconnected and open. When the latter was closed a falling off was found in the necessary velocity of break for given current. This is shown by the crooked line under IX. in Fig. 10. This is due to the absorption of energy by the secondary as was shown by the diminished electrometer readings for the primary oscillations.

When the secondary was connected to an electrometer and readings taken simultaneously with those of the primary electrometer, the maxima of electrometer readings for the two circuits were found to occur at the same time; *i. e.*, the induced potential in the secondary is a maximum, when the current in the primary has its greatest

[1] Loc. cit.

possible value consistent with no sparking at the break. This is equivalent to the well known fact that the secondary potential is greatest when the optimum capacity is used with the primary. Moreover, it was found that the greater the velocity of break the greater the possible secondary potential, because it enables the primary current to be made larger, or the capacity to be made less. This is in direct agreement with Lord Rayleigh's result.

Core. — Heavy cores of brass were used without perceptible effect on the necessary velocity of break. After Series VII. was obtained, bundles of iron wire were introduced into the core of the coil, and caused a marked falling off of the necessary velocity of break, as shown by the dotted line in Fig. 10. There was also a great falling off in the electrometer readings, probably due to the hysteresis loss and the lengthening of the period. According to a principle laid down by B. Walter[1] "the maximum potential for the secondary, excepting for losses of electric energy through break sparks, magnetic and dielectric hysteresis, and Joule's heat, is directly proportional to the maximum value of the potential of the primary." When the secondary is open, the velocity of break necessary to prevent sparking will be determined by the constants of the primary circuit and by the value of the initial current; and this velocity of break will permit of the highest possible secondary potential. When the secondary is closed, energy is absorbed from the primary, the necessary velocity of break will be less than for open secondary. The investigation will be complicated in this case by the introduction of a damping factor in the primary circuit.

Mercury Break. — In some early experiments a study of the mercury break was made. The break consisted of a blunt needle which was withdrawn at a definite rate from a well of mercury. Photographs were taken of the break, the illumination being that of its own spark. It was found that a column of mercury followed the needle for a short distance above the surface before separation took place. The velocity of break is thus not as great as could be obtained with a mechanical break going at the same rate, showing the decided advantage of the latter. No dependence of optimum capacity and velocity of break could be found except for very small

[1] B. Walter, Wied. Ann., 62, p. 300, 1897.

induced potentials. This we should expect, since a given point has a fixed discharge potential such that for higher values discharge will occur independently of the distance.

It was found that the capacity necessary to prevent a spark at the break was about one half as much when the needle was connected to the positive pole of the battery as was required when it was connected to the negative pole, so that much greater induced potentials could be reached in the former case than in the latter. This agrees with results which have been published by Dr. J. E. Ives.[1] It is probably due to the fact that the discharge potential for a negatively electrified point is less than for the same point positively electrified[2] and is a good point to be remembered by those using the mercury break with an induction coil.

Summary.

This work was undertaken in the hope of finding relations which were indicated in some experiments by Lord Rayleigh, existing between the constants of an oscillatory electric circuit, such as the primary of an induction coil, the initial current, and the velocity with which the contacts of the break are separated, when the conditions are just sufficient to prevent sparking at the break.

By neglecting the "lag" effect in the spark, a theoretical relation was found, and may be roughly expressed by saying that the velocity of break necessary to prevent a spark will be proportional to the frequency of oscillation and a linear function of the maximum induced potential, the latter implying a linear relation between velocity of break and initial current.

The necessary velocity of break was found experimentally to be:

1. A linear function of the initial current (see Figs. 7–10).

2. For given current, increasing in value in a manner consistent with the equation when the capacity is diminished (Figs. 6, 8, 10).

3. Diminishing according to the equation when the inductance is diminished (Fig. 9.)

The assumption that the lag of the spark might be neglected did not hold true. It was found that the spark constants, a and b,

[1] J. E. Ives, On the Asymmetry of the Mercury Break, PHYS. REV., 17, p. 175, 1903.
[2] Tamm, Annalen der Physik, 6, p. 259, 1901.

were too large, but diminished systematically as the time between the application of the potential and the passage of the spark increased, and would apparently reduce to the ordinarily accepted values if the time were great enough to allow the gas to become sufficiently ionized.

The results enable us to place a limit on the velocity of break necessary to prevent a spark at the break of an inductive circuit, if we know the inductance, capacity and initial current. The necessary velocity will be lessened by any condition which causes the absorption of energy from the primary, such as a closed secondary, hysteresis, heat losses, etc.

The method here developed seems to be well adapted to the study of the lag effect in the electric spark, or of the ionization produced by an electric field. A method in which use is made of constant potentials is likely to present great mechanical difficulties, especially as these results seem to show that the potential necessary to produce a spark across a given distance within the time t will have decreased from infinity to only e times its normal value in about 3.6×10^{-5} sec. The mechanical difficulties are reduced to a minimum in the present method, the main trouble to be expected lying in the interpretation of results. I hope soon to carry out similar experiments with the method much improved and with results covering a wider range for the purpose of getting data on the ionization produced by an electric field.

I wish to express my best thanks to Professor Arthur G. Webster, who has constantly aided me with suggestions, and to the authorities of Clark University for the facilities placed at my disposal.

July, 1904.

EXPERIMENTS ON RESONANCE IN WIRELESS TELEGRAPH CIRCUITS. PART IV.

By George W. Pierce.

X. The Direct Coupled Type of Sending Circuit.

IN the previous parts[1] of this research an account was given of experiments on the resonance conditions in wireless telegraph circuits of the "electromagnetically-connected" type. It is proposed in the present paper to present the results of similar investigations with other types of circuits.

The sending station in the set of experiments here described was kept constant throughout, and is represented diagrammatically in Fig. 30. It is seen to consist of an autotransformer instead of the tesla arrangement of the previous experiments. The high frequency oscillations in the sending antenna are produced by the discharge of a condenser through some of the turns of an inductance in the mast circuit.

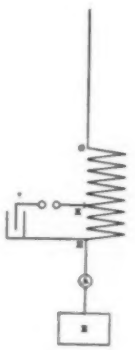


Fig. 30.

This form of sending circuit is employed in several of the commercial systems of wireless telegraphy at present in operation. Itseems to have been first proposed by Professor Ferdinand Braun of Strassburg, and is covered by German Patent No. 111,578, issued October 14, 1898, to the Gesellschaft für drahtlose Telegraphie. Braun's claim of priority in the use of this circuit is substantiated by Righi and Dessau,[2] Max Wien,[3] Fleming and various other writers on wireless telegraphy. For example, Fleming[4] says, "The second method, due to Braun, consists of attaching the aerial to some point on an oscillation circuit consisting of a condenser, an

[1] Physical Review, Vol. XIX., p. 196, 1904; Vol. XX., p. 220, and Vol. XXI., p. 367, 1905.

[2] Righi and Dessau, Telegraphie Ohne Draht, p. 416, 1903.

[3] Wien, Ann. d. Phys., 8, p. 686, 1902.

[4] Fleming, Pop. Sci. Monthly, June, 1903.

inductance coil and a spark gap, in series with one another, and charging and discharging the condenser across the spark gap so as to create alternations of potential at some point on the oscillation circuit. The length of the aerial must then be so proportioned * * * that it is resonant to this frequency."

Adjustment of Sending Circuit. — Evidently in order to produce a strong radiation of electric waves the different parts of the sending station must be properly proportioned. This adjustment may be made in several ways; by variation of the capacity of antenna, the capacity of the condenser, the whole inductance *GH*, or the part of the inductance in the condenser circuit *KH*.

For the present experiments the first three of these quantities were arbitrarily chosen and kept constant, and adjustment was made by moving the connector *K* to various points on the inductance. At each position the key in the primary of the charging transformer was closed and the current between the antenna and the ground was read on a hot-wire ammeter at *A*. The connector *K* was finally set at the position that gave the greatest current in the ammeter. This adjustment makes the sending station into a system with a strong oscillation of current in the antenna, which is the condition for a strong radiation of energy.

The dimensions of the parts of the sending station in their final adjustment had the following values:

The sending antenna, shown in Fig. 13 of Part II., extended to a height of 15.8 meters above the mast inductance. For the greater part of its length it consisted of four wires .208 cm. in diameter and 59 cm. apart.

The coil *GH*, Fig. 30, consisted of five turns of wire .208 cm. in diameter wound in a spiral 46 cm. in diameter with a pitch of 5.08 cm. The total inductance of this coil, measured on a Rayleigh's bridge, is 1.56×10^{-5} henries.

The part of the coil between the leads to the condenser, *KH*, consisted of 1.2 turns and had an inductance of $.151 \times 10^{-5}$ henries.

The condenser was made up of sheets of copper separated by miconite plates. Its capacity was not measured.

XI. Resonance Curves with the Direct Coupled Sending Station and Various Forms of Receiving Circuit.

Experiment XII. Tuning the Receiving Circuit with a Variable Inductance in the Antenna. — In many of the wireless telegraph plants in commercial operation the receiving station is tuned by varying an inductance placed above the instrument in series with the antenna. This is the method of the present experiment. Fig. 31 is a diagram of the receiving circuit. The receiving instrument represented at I is the high frequency dynamometer used in the previous experiments. The coil of the dynamometer has a resistance of 1.33 ohms and an inductance of 1.17×10^{-5} henries.

Fig. 31.

The tuning coil L is a variable inductance and consists of 51 turns of copper wire .208 cm. in diameter, wound on a vulcanite drum 13 cm. in diameter. The pitch of the windings is .42 cm. Variation of its inductance is made by a wheel contact moving along the wire as the drum is rotated. Values of the inductance for various positions of the movable contact are known from a calibration of the tuning coil by the aid of a Rayleigh's bridge. The inductance of the whole of the tuning coil is 16.5×10^{-5} henries. Any fraction of this inductance can be used.

The experiment consists in finding what inductance is required to be added by the tuning coil L in order to bring the receiving station to resonance with the sending station when various lengths of antenna are used at the receiving station. Quantitative readings were taken so as to give also the shape of the resonance curves and the intensity of the signals received.

A set of observations with this method of tuning are shown in the curves of Fig. 32. The different curves were obtained with different heights of the receiving antenna, the heights being designated by the numbers at the vertices of the curves. The abscissas of the curves are the inductance of the tuning coil; the ordinates are the deflections in centimeters all reduced to one scale, namely, the deflection that would be had if the receiving instrument should be set at one fiftieth its best sensitiveness.

Each curve has two distinct maxima, showing that the direct coupled sending circuit like the electromagnetically coupled circuit has two distinct periods of oscillation, as has been frequently pointed out.

Sharpness of Resonance. — The curves of Fig. 32 are seen to be very obtuse in comparison with those obtained with the arrangement of apparatus in the previous papers. For example, the first curve of the present series has a maximum for 1.8×10^{-5} henries of added inductance, which together with the inductance of the instrument makes the total inductance 2.97×10^{-5}. The deflection drops to half for a decrease of 26 per cent. or an increase of 60 per cent. of this value. This result compares very unfavorably with

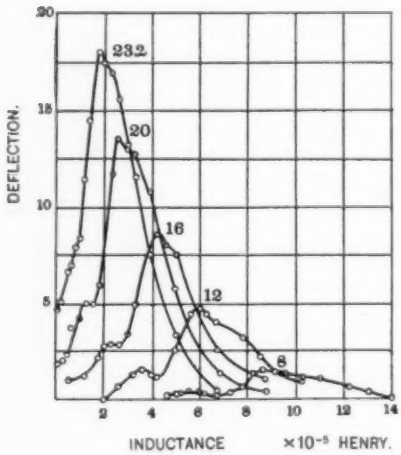


Fig. 32.

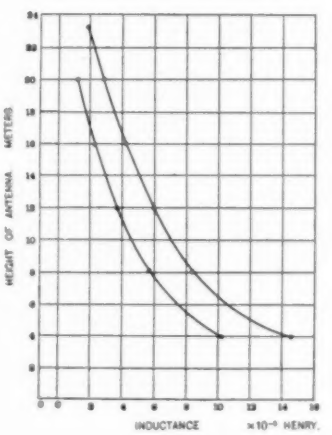


Fig. 33.

some of the curves of the previous research with the electro-magnetically coupled system, where a corresponding change of 5 per cent. or even 2.5 per cent. has caused the energy received to drop to one half. This loss in sharpness of resonance does not seem to be adequately compensated for by a gain in intensity of signals, for the deflections in the present experiment are only 1.4 times as great as those that were obtained in the case that gave the fall of energy to one half for a 5 per cent. departure from resonance. The power expended at the sending station was about the same in the two cases.

Relation Between Height of Antenna and Resonant Inductance in the Tuning Coil. — As the length of antenna is diminished in the above experiment, the inductance in the mast circuit must be increased in order to obtain maximum deflection with a fixed frequency of incident waves.

TABLE XIV.

Showing Inductance Required for Resonance. Height Varied. Incident Waves Constant.

Height, Meters	L' Henry.	L'' Henry.	l' Meters.	l'' Meters.
23.2	1.8×10^{-5}		4.08	
20	2.65	1.2×10^{-5}	5.30	2.86
16	4.2	2.35	7.35	5.30
12	5.0	3.6	9.60	6.50
8	8.5	5.6	12.25	9.00
4	14.6	10.3	18.80	14.30

Table XIV., taken from the curves of Fig. 32, shows the values of the inductance of the tuning coil required with various heights of antenna in order to obtain the maximum deflection. L' is the inductance that produces the strong maximum, and L'' the weaker maximum ; l' and l'' are the length of wire in these two inductances respectively. The two maxima arise in the existence of two periods of the incident waves.

The values of L' and L'', recorded in Table XIV., are plotted in Fig. 33, in which the two curves relate to the two periods. These are empirical curves. The distance OO' is the inductance of the instrument. The fact that the curves of Fig. 33 are not straight lines shows that the problem of the propagation of electric waves in wire wound in a coil is less simple than the problem of its propagation in a straight wire. In the case of the non-magnetic straight wire in air the self inductance per unit of length is inversely proportional to the capacity per unit length, so that the velocity of propagation in the wire, as derived from the wave equation, is constant and is equal to the velocity of light. This deduction from the theory of electric waves has been amply verified by experiments. In some other simple cases, *e. g.*, the case of two parallel wires or two coaxial cylinders, the same relation of inductance to capacity holds and the velocity of the waves in such conductors is the velocity of light. On the other hand, in the case of the solenoid here

used as a tuning coil, in order to give curves of the shape of those of Fig. 33, the velocity of the waves is different from the velocity of light and furthermore the velocity of the waves in the solenoid is different in different parts of the solenoid. This is evident if we recall that with a resonant receiving circuit the time for the wave to travel from the top of the receiving antenna to the ground is one fourth the period of the sending circuit. When we shorten this time by cutting off antenna we must increase it equally by adding inductance if the circuit is to remain resonant.

It is seen from the curves that for *equal* decrements of the length of antenna, which are proportional to decrements in time, we need to make *unequal* increments of the inductance, which are, therefore, not proportional to the time. In a similar way, by plotting length of antenna against length of wire in the solenoid (curves not here given), it may be seen that lengths on the solenoid are not proportional to the time for waves to traverse them ; that is the velocity of the waves on the wire of one part of the solenoid is different from their velocity on the wire of another part of the solenoid. In terms of the theory of electric waves in a conductor this is equivalent to saying that the inductance per unit of length of the solenoid is not inversely proportional to the capacity per unit of length of solenoid. This same result has been reached by Drude,[1] in a masterly theoretical and experimental research on the natural period of electric oscillation of solenoids with antenna attached.

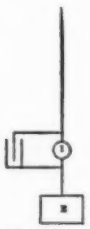


Fig. 34.

The wave-length here used is determined in the next experiment in which the conditions prove to be simpler.

Experiment XIII. Tuning the Receiving Circuit by a Variable Capacity Shunted About the Instrument. — The form of receving circuit employed in this experiment is shown in Fig. 34. When the antenna has not sufficient capacity to be in resonance with the incident waves, additional capacity is introduced in the form of a variable condenser shunted about the coil of the receiving instrument. The condenser used is one of the calibrated air condensers described in Part I.

Evidently as the length of antenna (a single wire .208 cm. in diameter) is diminished, the shunted capacity must be increased. A

[1] Drude, Ann. der Phys., 9, p. 293, 1902, and 11, p. 957, 1903.

set of the observations obtained is presented in the curves of Figs. 35 and 36. (In order to bring out details the curves of Fig. 36 have their ordinates magnified [1] twenty-five times with reference to Fig. 35.) The numbers at the vertices of the curves designate

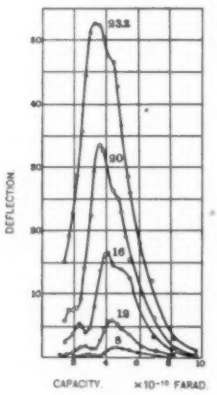


Fig. 35.

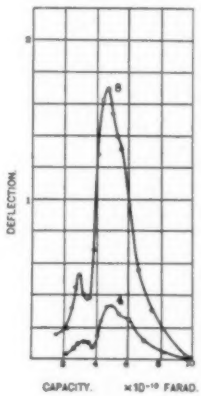


Fig. 36.

heights of antenna in meters. The abscissas are capacity of shunt, and the ordinates are the deflections, reduced to centimeters at one fiftieth the best sensitiveness of the receiving instrument.

These curves bear a marked resemblance in form to the curves of Fig. 32. The irregularities near the summits, possessed in common by the two sets of curves, evidently belong to the wave produced at the sending station and are not characteristic of the receiving station.

Other experiments have shown that these irregularities may be eliminated by better adjustment of the parts of the sending circuit to resonance with each other.

Sharpness of Resonance. Hysteresis Loss in Dielectrics Other Than Air. — As to sharpness of resonance the curves obtained by tuning with the aid of the shunted capacity are about equivalent to those obtained by tuning with inductance in series. It might be remarked in passing that glass condensers and mica condensers, when used in place of the air condenser at the receiving station, gave about the same sharpness of resonance and nearly the same intensity of signals as the air condensers.

[1] Small readings are made with as great accuracy as large readings, because the small readings are taken as large deflections with increased sensitiveness of the instrument.

To show the relative merit of the two dielectrics the following readings of deflections, Table XV., are taken at random from among the observations in which both glass and air condensers were used as shunts about the receiving instrument.

TABLE XV.

Showing Relative Merit of Glass and Air as Dielectrics.

	Air.	Glass.	Glass / Air.
Deflection.	7.0	6.6	.95
	8.6	7.9	.94
	11.3	10.6	.95
	39.0	37.7	.97
	54.0	49.5	.91
			Mean .94

Whence it is seen that in comparison with air the loss of energy by hysteresis or leakage when glass is used as dielectric in the condenser at the receiving station amounts to about six per cent.

Experiments were also made with mica and waxed silk as dielectrics, but the comparison with the air condensers in those cases were not direct, so the numerical data are not reported. It appeared, however, that with the present form of receiving circuit Fig. 34, condensers of either of these substances or of glass may be used in place of air condensers at the receiving station without great loss of energy. An exhaustive study has not yet been made with a variety of dielectrics, and the result here given should be regarded merely in the nature of a preliminary note.

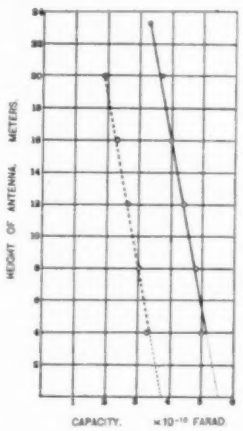


Fig. 37.

Relation between the Length of Antenna and the Resonant Capacity of the Condenser Shunt. — In Fig. 37, height of antenna is plotted against the shunt capacity required for resonance. The relation is more simple than the relation of resonant inductance to height in the previous experiment, and is represented by two straight lines, corresponding respectively to the two periods of the incident waves.

The straight line relation is taken to indicate that as capacity is cut off from the antenna equal capacity must be added in the condenser about the instrument. If we proceed on this theory it is easy to determine the two wave-lengths sent out from the sending station.

The straight lines in Fig. 37, when produced, cut the axis of abscissas at the points $C' = 5.5 \times 10^{-10}$ and $C'' = 3.7 \times 10^{-10}$ farads. These quantities give respectively the values of capacity that shunted about the instrument *with zero antenna attached* form the two cases of closed circuits in unison with the sending station. The period of such closed circuits is given by the formula

$$T = 2\pi\sqrt{LC}. \tag{1}$$

Knowing L, the inductance of the instrument, and the two values of C (C' and C'') we may obtain the two periods of the sending circuit by substituting these quantities in equation (1).

$$T' = 2\pi\sqrt{5.5 \times 1.17 \times 10^{-15}} = 51 \times 10^{-8} \text{ sec.},$$

and

$$T'' = 2\pi\sqrt{3.7 \times 1.17 \times 10^{-15}} = 43 \times 10^{-8} \text{ sec.}$$

Whence the two wave-lengths of the sending system are

$$\lambda' = vT' = 153 \text{ meters}, \quad \text{and} \quad \lambda'' = vT'' = 129 \text{ meters}.$$

Special Case of the Electromagnetically-connected Receiving Circuit. — The straight-line relation shown in Fig. 37 may be derived theoretically by treating the circuit with capacity shunted about the receiving instrument, Fig. 34, as a special case of the electromagnetically-connected receiving circuit given in Fig. 10, Part II. In Part II. (cf. eq. (4), p. 246, Vol. XX.) it is shown experimentally and theoretically that the condition for resonance is

$$(L_3L_4 - M^2)\omega^4 - \left(\frac{L_3}{C_4} + \frac{L_4}{C_3}\right)\omega^2 + \frac{1}{C_3C_4} = 0. \tag{2}$$

With the arrangement of apparatus in the present experiment

$$L_3 = L_4 = M.$$

Whence, the condition for resonance becomes

$$\omega^2\left(\frac{L_3}{C_4}+\frac{L_3}{C_3}\right)=\frac{1}{C_3C_4} \tag{3}$$

or

$$C_3+C_4=\frac{1}{L_3\omega^2} \tag{4}$$

In this equation C_3 is the capacity of the receiving antenna and is approximately proportional to its length; C_4, the capacity of the condenser shunted about the instrument; L_3, the inductance of the instrument; and ω the angular velocity of the incident waves. Equation (4) is the linear relation shown in Fig. 37 and discussed in the preceding section.

If we let $C_3 = 0$, and make use of the corresponding values of C_4, we obtain equation (1) and confirm the above calculation of wave-lengths.

This remark is here inserted as an example of the manner in which the whole theory of the direct-coupled system of wireless telegraphy may be regarded as a special case of the electromagnetically-connected system studied in Parts I., II. and III. of this research.

Capacity of Receiving Antenna. — In Fig. 37 the linear relation between the height of antenna and the capacity that must be shunted about the instrument to produce resonance gives a simple method of obtaining approximately the capacity of a given length of the antenna. For example, the right hand straight line shows that 23.2 meters of a single wire .208 cm. in diameter with an added condenser capacity of 3.4×10^{-10} farads gives the same period of oscillation as 4 meters of wire with an added capacity of 5.0×10^{-10} farads. That is, when we cut off 19.2 meters of antenna, we must add 1.6×10^{-10} farads as a compensation, and if we neglect the fact that the capacity of the wire is distributed instead of localized, we may say that 19.2 meters of this particular wire has a capacity of 1.6×10^{-10} farads. This would give the value 8.35×10^{-12} farads as the average capacity of one meter of the antenna, a value that agrees satisfactorily with the value 8.2×10^{-12} computed from the usual formula for the capacity per unit length of a straight conductor

$$K=\frac{l}{2\ln l/r},$$

in which r is the radius of the wire in centimeters, $l = 100$ cm., and ln means natural logarithm.

The parallelism of the two straight lines of Fig. 37 arises in the fact that both lines must give the same value for the capacity of a given length of antenna.

Relation of Integral Current Received to Height of Receiving Antenna as Determined by Experiments XII. and XIII. — As the height of the receiving antenna is diminished the amount of energy received falls off rapidly. The curves of Fig. 38 give the deflection plotted against height of antenna. The values on the lower curve *A* were obtained when the receiving circuit is brought to resonance by putting proper inductance in the mast circuit. The values on the upper curve *B* are the corresponding deflections when the receiving circuit is tuned by the capacity in shunt. All the deflections are on the same scale and were taken as alternate readings so as to eliminate uncertainties arising from any slow changes that might occur in the sending station. It is seen that the latter method of tuning gives larger deflections. The amount of the advantage the shunt capacity has over the tuning coil is different for different heights. The advantage increases with increasing height, and in the particular case studied, when the height of antenna is 23.2 meters, the deflections with the capacity in shunt are three times as great as the deflections with the tuning coil in series.

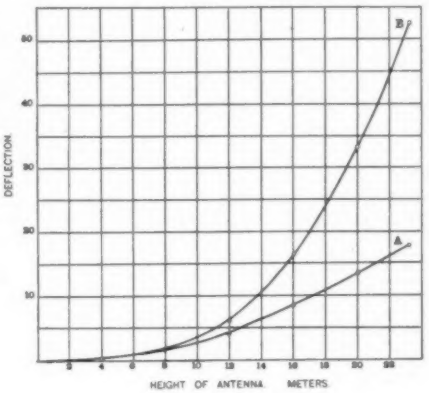


Fig. 38.

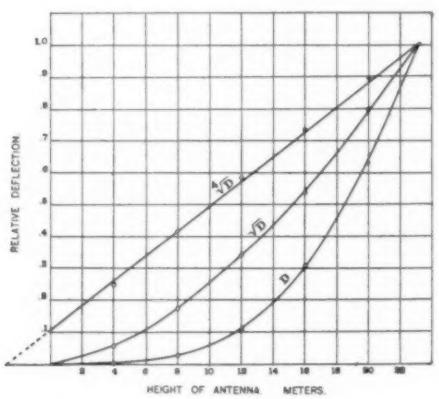


Fig. 39.

In order to examine further the relation of height of antenna to

current in the receiving instrument the curve B of Fig. 38 is plotted in Fig. 39 with the ordinates all changed by a constant multiplier chosen to make the largest deflection unity. Together with this the square roots and the fourth roots of the deflections are also plotted against height of antenna. It is seen that the relation of the fourth root of the deflections to the height of antenna is represented approximately by a straight line, which cuts the axis of ordinates at 2.75 meters. The heights are here measured from the coil of the receiving instrument. If the straight line relation permits of slight extension, the negative intercept 2.75 should indicate the height of the instrument above the point of effective ground. This interpretation seems not unlikely when we take into account the fact that the wire lead from the instrument to the surface of the earth is about two meters long; this two meters is also exposed to the waves.

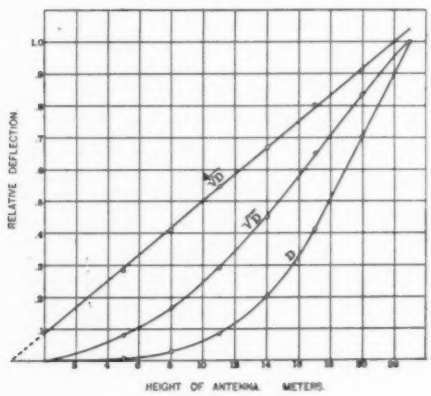


Fig. 40.

A search among earlier data in the observer's notebook disclosed that a previous experiment under different conditions, but with the same method of tuning, gives confirmatory data, which are plotted in Fig. 40 and which show also the fourth root of the deflections proportional to height. The intercept two meters in Fig. 40 is slightly less than that of Fig. 39, but the difference is not much greater than the error in determining the intercepts by lines drawn through the points.

In Part II. it was deduced theoretically and proved experimentally that the deflections of the instrument are proportional to the square of the current through it; therefore, the relation given by the curves of Fig. 39 and Fig. 40 may be stated as follows:

I. *With a constant sending circuit and a constant source of waves the current received in a circuit of the form of Fig. 34, brought to resonance by a capacity in shunt, is proportional to the square of the height of the receiving antenna measured from the "effective ground."*

In applying this law the point of effective ground may be determined graphically if the current received from *two* heights is known. As an approximation the effective ground may be taken as coincident with the surface of the earth.

The law applies only to an approximately vertical receiving antenna, and to the capacity-shunt method of tuning. Its application to that case seems rational, because the current through the instrument is proportional to the impressed electromotive force and to the capacity of the antenna; both the capacity of attenna and electromotive force are proportional to the height of antenna: therefore, current is proportional to the square of the height of antenna.

When the series inductance is used in tuning, a different relation is obtained. The relative deflections in this case, taken from curve *A*, Fig. 38, and plotted with the largest deflection put equal to unity, are shown in Fig. 41. In this figure the square roots and fourth roots of the deflections are also plotted against height of antenna. Instead of the fourth roots of the deflections falling on a straight line as they did in the previous case, the square roots with this method of tuning are approximately in linear relation to the heights of antenna. That is

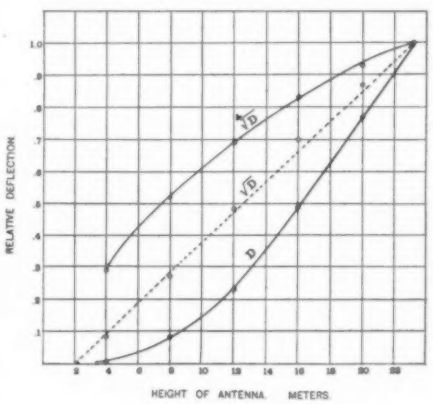


Fig. 41.

II. *The current in the receiving instrument is proportional to the height of antenna*, when the circuit is brought to resonance by added inductance in series with the antenna.

This statement is probably only an approximation, as seen from the fact that the points obtained as square roots of the deflections, instead of lying well on the dotted straight line in Fig. 41, apparently follow a wavy line about it. Also the dotted line cuts the x-axis at 2 meters, so that in applying the rule, 2 meters must be subtracted from the height of antenna measured from the receiving instrument.

The relations I. and II. here stated for the dependence of the current in receiving instrument upon height of receiving antenna may fail of verification when tested with greater height [1] of antenna, but the results are interesting in that they show entirely different laws for the two methods of tuning.

Experiment XIV. Tuning with the Instrument in a Direct Coupled Side Circuit at the Receiving Station. — Measurements were made with the form of receiving circuit shown in Fig. 42. The points M and D are contacts movable along the tuning coil used in Experiment XIII. The inductance between M and B is referred to as the inductance in the mast circuit. The inductance in the part of the coil between D and B is the added inductance in the instrument circuit. The results obtained are shown in the curves of Fig. 43.

Fig. 42.

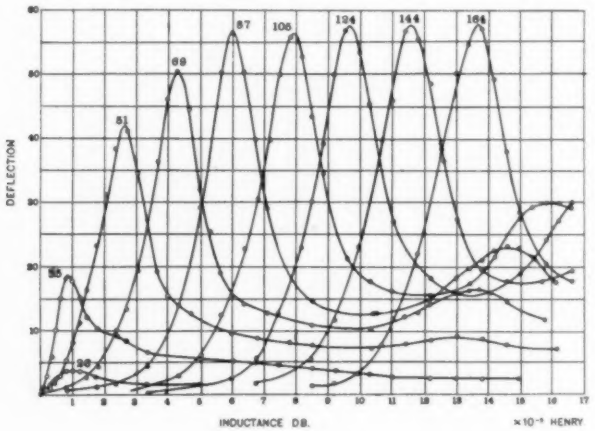


Fig. 43.

In taking any one of the curves the position of M was kept fixed at the values written at the vertices of the curves, which when multiplied by 10^{-6} give the inductance in the mast circuit. Deflections were then taken for various positions of the contact D, that is for various values of the inductance in the instrument circuit.

The height of the receiving antenna, 23.2 meters, remained unchanged during the experiment.

[1] In the experiments the height of antenna extended from 4 meters to 23.2 meters, that is to a little more than half of the quarter wave-length.

The meaning of the curves of Fig. 43 is made clear by a reference to Fig. 44, in which inductance in the mast circuit is plotted against the total inductance required in the instrument circuit to produce resonance. The former values are the numbers written at the vertices of the curves; the latter are the ordinates belonging to those respective vertices plus the inductance of the instrument.

Fig. 44 is a straight line and the corresponding values of the inductance in the mast circuit and the inductance in the instrument circuit have a common difference of 2.60×10^{-5} henries as given by the intercept on the axis of ordinates. This difference is evidently the inductance between the point *M* and the earth by the path *MDIE*, Fig. 42. The experiment shows that for resonance the inductance of this path must have the fixed value 2.60×10^{-5} henries. By a separate experiment the value 2.62×10^{-5} henries was found for the inductance of this path when an open circuit was made at the point *B*. We have in this last case, however, the simple form of circuit examined in Experiment XII.

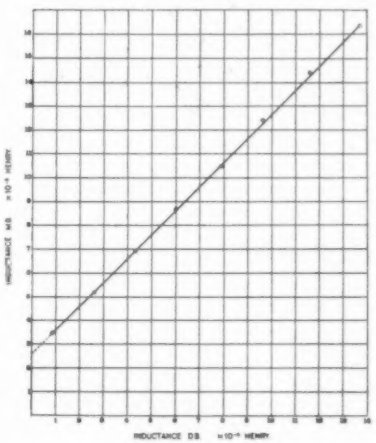


Fig. 44.

From these facts it appears, as should be seen by a glance at the circuit of Fig. 42, that the present experiment is dealing with the simple circuit of Fig. 31, with, however, an inductive[1] shunt inserted about the instrument. *The presence of the inductive shunt does not modify the resonance conditions. It simply diminishes the amount of energy obtained in the instrument.*

After the shunt has reached the value 6.0×10^{-5} henries its effect in this respect is small, so that the curves to the right of the curve with maximum ordinate 6.0, Fig. 43, have approximately constant amplitude, which is about 90 per cent. as great as the deflection obtained with the open circuit at *B*.

[1] The inductive shunt is subjected to inductive action from the current in the rest of the coil.

XXII. EXPERIMENTS TO TEST THE IMAGE THEORY OF THE ACTION OF THE GROUND.

Grounded and Ungrounded Circuits. — The preceding experiments have all made use of grounded circuits at the sending and receiving stations. A number of wireless telegraph plants are in operation in which the circuits are not grounded. I am not aware that quantitative measurements have hitherto been made comparing the efficacy of the ungrounded circuits with the grounded.

In the experiments described below basis is afforded for at least a tentative comparison, although the experiments were undertaken primarily to illustrate the manner in which the ground acts, and the results are to be viewed chiefly as a test of the image theory of the grounded circuits.

Image Theory. — According to this theory a perfect ground introduces into the circuit a point of zero fluctuation of potential and, therefore, maximum fluctuation of current at the place where the ground is attached. The top of the antenna, on the other hand, is a point of zero current and maximum fluctuation of potential. In addition to these two end conditions the current and potential must obey certain differential equations throughout the length of the antenna. So far as concerns the part of the circuit above the earth, these conditions are the same as should obtain if the circuit instead of being grounded should be electrically mirrored in the earth. That is, the oscillation in the grounded circuit is simply the oscillation in one half of a symmetrical Hertz oscillator made up of the existing aerial system and its image, the image however contributing nothing to the radiation or reception of the waves.

To examine this view somewhat more in detail, let us confine our attention to the receiving station, and suppose we had there simply a rectilinear conductor isolated in space and placed parallel to the electric displacement of the incident waves. Let the length of the straight-line conductor be so chosen that its natural period of electric oscillation is equal to the period of the waves. The conductor would then be resonant with the incident waves. For a given strength of waves there would be a maximum current back and forth at the center of the conductor because on either side of the center there is the electrostatic capacity of half the conductor. At a point half way between the center and the end of the conductor

the current back and forth would be less because out beyond this point the capacity is only the capacity of one fourth the conductor; while at the end of the conductor the current would be zero because there is no capacity beyond into which the current can flow.

We must now suppose that whatever current flows into one half of the conductor flows out of the other half; so that the potential at points in one half of the conductor rises while the potential of the other half falls. Thus the potential at the middle does not change, and the fluctuations of potential increase as we go toward either end. If we could introduce a current-reading detector into the circuit without disturbing the conditions, the instrument would give a maximum reading when placed at the center of the receiving conductor, that is, at the point where the fluctuation of potential is zero. Suppose with such an instrument in the circuit we should cut away one half the conductor; the reading would become zero. If now a capacity is attached to the instrument in place of the removed conductor, some current would flow between the straight wire and the capacity and register in the instrument.

If the capacity attached were very large (*e. g.*, the earth), the point of zero fluctuation of potential would again be brought near the instrument and we should have the same current as when the conductor was made up of two parts symmetrical about the instrument. Since, however, with the earthed circuit the antenna exposed to the radiation is only one half the original rectilinear conductor, to make the conditions identical we should have considered the other half of the original symmetrical system to be shielded from the action of the waves.

It is by reasoning of this kind, though usually expressed in mathematical form, that the theory is developed that the action in the grounded circuit with the receiving instrument near the ground is the same as it would be if instead of being grounded it should be symmetrically duplicated below the surface of the earth.

Of course, in actual systems the grounding is not perfect, and therefore, the symmetrical image gives only approximately an equivalent system.

This theory is submitted to tests in the experiments that follow, by comparing resonance curves with various forms of grounded

circuits with the corresponding curves taken with an *image circuit* in the place of the ground.

Experiment XV. The Aerial Circuit and its Image Tuned by Variable Inductance. — In this experiment the sending station was the same as that employed in the preceding experiments (Fig. 30) and was constantly grounded. The arrangement of apparatus at the receiving station is shown in Fig. 45. By means of a switch at *S* the ground could be thrown off and replaced by metallic parts duplicating the aerial system. The duplicate is, however, not an exact image of the aerial system, because the second antenna had to be run horizontally instead of straight down.

The horizontal wire was made equal in length to the vertical antenna, 23.2 meters, and was supported about 1 meter from the

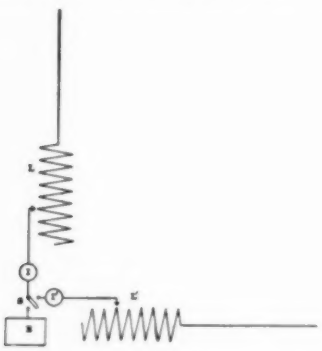


Fig. 45.

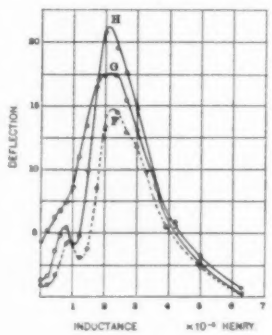


Fig. 46.

ground by cords attached to posts. In series with the horizontal wire was a variable inductance *L'* duplicating the tuning coil *L* and a small coil *I'* of fine wire, wound to duplicate the coil of the receiving instrument.

Curves giving the results of the experiment are shown in Fig. 46. For Curve *G* the grounded circuit was used, and readings of deflections were taken for various values of the inductance of the tuning coil *L*, Fig. 46; the deflections are plotted against values of *L*.

For curve *H* the switch was thrown so as to connect the aerial system with the horizontal system, Fig. 45, instead of with the ground, and deflections were taken for various values of *L* and its duplicate *L'* kept identical in value and varied together. In curve *H* the deflections are ordinates and the common values of inductance *L* and inductance *L'* are abscissas.

Discussion of Results. — The two curves *G* and *H*, Fig. 46, are seen to have their maxima for the same abscissa. That is, a given value of inductance, 2.1×10^{-5} henries, gives a maximum deflection in the grounded circuit. To obtain a maximum with the duplicated system the same inductance 2.1×10^{-5} henries must be used in both the vertical circuit and its horizontal duplicate. This result is a confirmation of the image theory.

It is interesting to note that the *deflection* (*current square*) *is about 20 per cent. larger with the duplicated system than with the grounded system* — a fact that may be accounted for by supposing a higher resistance in the grounded system than in the wholly metallic system.

The resonance curve *H* taken with the wholly metallic circuit is somewhat different in shape from the curve *G* with the grounded circuit. The two curves agree well in shape for points to the right of the maximum, but for points to the left the curve with the grounded system slopes away more gradually and does not show the secondary maximum that is so marked with the duplicated system. That there is, however, a tendency to a secondary maximum in the former curve appears in a slight bulge in the curve and also in the fact that secondary maxima are shown by other members of the same family given in Fig. 32. The greater prominence of the secondary maximum of the curve *H* goes with the general greater sharpness of the curve as the result of the diminution of resistance accompanying the substitution of the metallic "image" for the ground.

After a number of preliminary tests in which the horizontal circuit was not made so close a duplicate of the aerial parts, the result shown in Fig. 46 seemed a rather striking agreement with the image theory.

In the effort to find a more convenient artificial ground than that supplied by the metallic duplicate of the aerial system the curve *F* was obtained, with the duplicate antenna wound around the house of the receiving station instead of extending out horizontal. In this case the capacity balancing the antenna was in the form of a rectangular spiral three meters square with a pitch of forty centimeters. This was a convenient arrangement, but was accompanied by a loss of some 20 per cent. of the energy received with the grounded circuit. Other arrangements, for example, a large number of short wires in the form of a horizontal fan, supported about one meter above the earth, gave about the same energy as that obtained with

the metallic duplicate of the aerial circuit, that is 20 per cent. larger than the connection to earth.

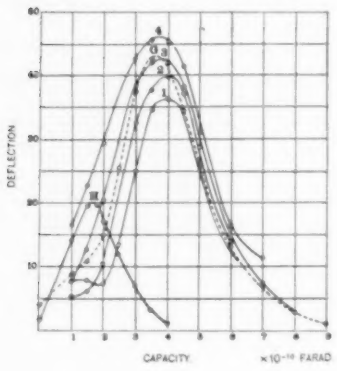


Fig. 47.

Experiment XVI. Tuning With Capacity in Shunt. — In further test of the image theory of the action of the ground, experiments were made with the capacity-shunt form of aerial circuit shown in Fig. 34. By throwing a switch this circuit could be grounded or completed by a metallic portion more or less nearly an image of the aerial system.

Curves are given in Fig. 47. The dotted curve *G* was obtained with the use of the ground; deflections are plotted against capacity about the instrument.

In taking the curve *H*, the ground was thrown off and a horizontal system was substituted for the ground. The horizontal system consisted of a duplicate of the coil of the instrument and a horizontal antenna 23.2 meters long. To have an exact duplicate there should be a variable condenser about the receiving instrument and an equivalent variable capacity about the duplicate of the instrument. As two such capacities were not easily available, instead of this arrangement the one variable condenser was put about both the instrument and its image. The curve *H* was obtained, in which deflection is again plotted against capacity of the shunt condenser. The maximum of *H* falls at a capacity one half the capacity for a maximum with the grounded circuit, *G*. This result confirms the image theory; for the condenser about the instrument and the image of this condenser in the duplicate system would be in series and therefore have together a capacity one half as great as the capacity for resonance with the grounded circuit.

In this experiment the deflections with the ungrounded circuit are a little less than half as great as the deflections with the grounded circuit. It is conjectured that this is due to the fact that our ungrounded circuit was not exactly a duplication of the original grounded system as may be seen from an examination of a diagram (not given) of the two arrangements. In the duplicated system a

path is lacking for the current to go through the image capacity and then through the instrument.

Experiment XVII. Aerial System with Capacity Shunt; Image with Series Inductance. — In this experiment an aerial system with capacity shunted about the instrument (Fig. 34) could be grounded or could be connected to a horizontal system with variable inductance in series (Fig. 45). The curve *G*, Fig. 47, is a resonance curve with the grounded system. The curves 1, 2, 3 and 4 were taken with the ground replaced by the horizontal member, consisting of horizontal wire 23.2 meters long with various values of inductance in series with it, as follows:

Curve No.	Inductance in Horizontal, Henries.
1	20×10^{-6}
2	27×10^{-6}
3	33×10^{-6}
4	39×10^{-6}

From the curves it is seen that the artificial ground that comes nearest to replacing the actual ground is the arrangement that gives curve 3. The horizontal portion of the circuit in this case is the same as the horizontal circuit and also the aerial circuit in experiment XV., for the common inductance in that case was 2.1×10^{-5} henries, which together with the duplicate of the instrument made a total inductance of 3.27×10^{-5} in each part of the circuit, which is in good agreement with the total inductance in the horizontal for curve 3, viz. 3.3×10^{-5} henries.

It should be remembered, however, that the curve 4 with greater inductance in the horizontal gives a larger deflection than that of curve 3 because with the greater inductance the point of maximum current is shifted into the instrument instead of being at the ground.

Experiment XVIII. Quarter Wave Ground. Determination of Wave-length. — The most interesting of the experiments confirmatory of the image theory of the action of the ground was made by replacing the ground by a straight horizontal wire of which the length could be varied. Resonance was obtained when this wire had the length of one fourth the wave-length.

Both the capacity-shunt and the inductance-series forms of aerial circuit were employed. These circuits were grounded and the capacity or inductance set for resonance. The ground was now

disconnected and a long horizontal wire was attached in its place This threw the system out of resonance as shown by a small deflection. Wire was cut off the horizontal in steps and readings were taken. These readings increased as the horizontal was shortened up to a certain length and then decreased with further shortening of the horizontal. The results are plotted in Fig. 48. The curve *A* is with the inductance-series aerial ; the curve *B* with the capacity-shunt aerial. The deflections were taken alternative with deflections with the circuit grounded and the ratio of the deflections with artificial ground to deflections with actual ground are plotted as ordinates against the length of horizontal as abscissas.

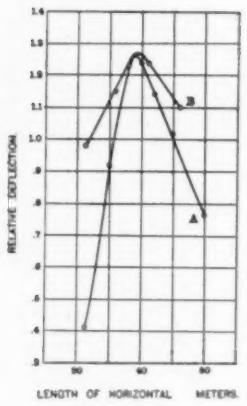


Fig. 48.

It is seen that in the experiments with both forms of aerial circuit the horizontal wire must have a length between 38 and 39 meters for a maximum deflection. If this is the quarter wave-length as the theory proposes, we should have for the wave-length a value between 152 and 156 meters, which is in excellent agreement with the value 153 meters found independently and by a different method in Experiment XIII.

Experiments XV., XVI., XVII. and XVIII. taken together show good agreement between the observations and the predictions from the image theory of the action of the ground. They also show that it is a simple matter to balance the aerial at the receiving station with an artificial ground that will increase the reception of energy (current square) by twenty per cent. or twenty-five per cent. over a good actual ground.[1]

JEFFERSON PHYSICAL LABORATORY,
HARVARD UNIVERSITY, CAMBRIDGE, MASS.
October 17, 1905.

[1] The ground at each station consisted of three copper plates and a piece of iron pipe. The copper plates 60 cm. square by .3 cm. thick, were embedded in the earth 270 cm. below the surface. The pipe, 3 cm. in diameter and 180 cm. long was driven in until flush with the bottom of the holes containing the plates, and therefore reached to a depth of 450 cm. below the surface.

THE EFFECT OF PRESSURE ON SURFACE TENSION.

By C. J. Lynde.

THE following article is an account of an investigation into the effect of pressure on the surface tension of the surface between two liquids.

Method. — Briefly, the method was to place the heavier of the two liquids in a U-tube, one arm of which was 1 cm. in diameter and the other a capillary tube; this was immersed in the lighter liquid, a pressure of from 1,000 to 6,000 lbs. per sq. in. was applied, and the change in the height of the meniscus in the capillary tube was observed by means of a micrometer microscope.

Apparatus. — The apparatus consisted essentially of: (1) A U-tube with one arm a capillary tube (Figs. 1 and 2). (2) A compression chamber, with plunger, screw, and gauge (Figs. 3 and 4). (3) Two windows opposite each other in the compression chamber, through which the movement of the meniscus could be observed.

The U-tubes (Figs. 1 and 2) were 14 cm. long, the larger upper tube being 9 cm. long and 1 cm. in internal diameter; the smaller tube connecting with the capillary tube was 5 cm. long and 0.5 cm. in internal diameter. The bore of the capillary tube varied for the different observations. Their diameters are given with the following tables.

The compression chamber *C* (Figs. 3 and 4) was of nickel steel 7.5 cm. square and 37 cm. long, the bore being 1.9 cm. in diameter. It was divided into two parts at *a* (Fig. 4) for the admission of the U-tube.

In the lower half, *w. w.* represent two cylindrical glass windows, 14 mm. deep and 14 mm. in diameter, cemented into hollow nickel steel plugs. The plugs were 33 mm. long, and 50 mm. in diameter at the head, and had a conical-shaped opening through the middle, which was 22 mm. in diameter at the head and 5 mm. in diameter at the windows. The upper half of the chamber screwed into

the lower half, and a soft copper washer *A*, 2 mm. thick, served to make the juncture tight.

B is a large plug 48 mm. in diameter at the head and 50 mm. deep, which screws into the chamber. It has a hole through the center, through which the plunger passes.

P is a glass-hardened steel plunger 143 mm. long and 11.5 mm. in diameter.

D is a thin brass collar turned so as to fit the plunger accurately and so arranged (see Fig. 4) that the greater the pressure the tighter it holds.

S is the screw which gives the downward motion to the plunger; it is 105 mm. long and 15 mm. in diameter.

G (Fig. 3) is a gauge reading as high as 20,000 lbs. per sq. inch.

M is a micrometer microscope with focal length of 6 cm. and 11 divisions on the micrometer scale = 1 mm.; 100 divisions on micrometer head = 1 division of micrometer scale.

METHOD IN DETAIL.

Mercury and Water. — The U-tube (Fig. 1) was cleaned carefully before each set of observations, by boiling it in strong sulphuric acid and dropping fragments of potassium bichromate into the liquid, so that the gas liberated passed up through the capillary tube; then it was boiled in distilled water, then rinsed with cold distilled water.

The mercury was triple-distilled and had been passed through dilute nitric acid solution.

The distilled water was freshly boiled and cooled. The mercury was poured into the U-tube, then placed under water and some of it forced out through the capillary tube, so that air, if present, was removed and a fresh water mercury surface secured.

The U-tube was then placed in the compression-chamber and adjusted so that the meniscus was in the field of view of the windows *w*, *w*.

The chamber was closed, filled with water, and the plunger and compression-screw adjusted.

The method of observation in all cases was to set the movable cross-hairs of the micrometer microscope on the meniscus, and read

its position. An assistant put on the pressure; the new position was read, the pressure was let off, and the position to which the meniscus returned was read.

Each individual reading in the tables is a mean of a movement with pressure and a return-movement when the pressure was released.

In the early experiments with mercury and water it was impossible to observe any movement of the meniscus, and even when the

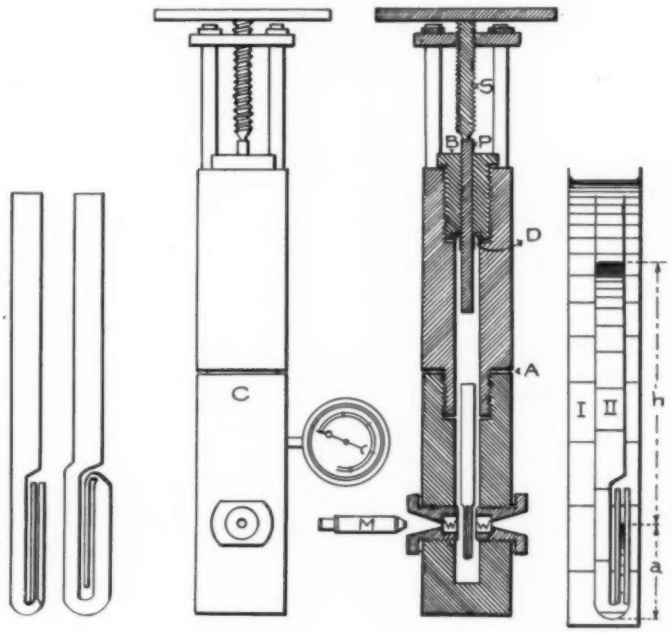


Fig. 1. Fig. 2. Fig. 3. Fig. 4. Fig. 5.

U-tube was taken in the hand and tilted from side to side the movement was slow and irregular. This was thought to be due to the formation of an oxide of mercury at the surface of contact. The difficulty was overcome by adding nitric acid to make the water a 3 per cent. acid solution.

The difference in level of the mercury columns h was measured by means of a cathetometer.

Method of Calculation.

Let T = surface tension of the contact surface.

θ = angle of contact.

h = difference in height in cm. of the columns of liquid II.

r = radius of capillary bore.

σ_1 = density of liquid I.

σ_2 = density of liquid II.

a = depth of liquid II. below meniscus.

Then

$$T \cos \theta = \frac{hr}{2}(\sigma_2 - \sigma_1).$$

In this equation each term may vary with the pressure.

The change in r can be neglected because, according to experiments of Amagat,[1] the linear compression of glass is .00000075 per atmosphere. The highest pressure used in this investigation was in the neighborhood of 400 atmospheres and this would produce a change in r of only .03 per cent., while the observed changes were all over 1 per cent. for this pressure.

From the experimental evidence at hand it appears that the contact angles of the surfaces employed in the present investigation are either 180° or in the neighborhood of 180°, so that a variation of θ with pressure will cause but a small variation in $\cos \theta$, and in the present discussion this variation has been neglected. To determine rigorously what effect pressure produces on $\cos \theta$ will require a further investigation.

The variables then are T, h, σ_1 and σ_2. The changes in σ_1 and σ_2 may be calculated from the coefficients of compressibility and the pressure. The change in h is that upon which the observations are made. From these the change in T is calculated in the following way:

Differentiating Equation 1 as to P

$$\frac{\delta T}{\delta P} \cos \theta = \frac{\delta h}{\delta P} \cdot \frac{r}{2}(\sigma_2 - \sigma_1) + \frac{hr}{2}\left(\frac{\delta \sigma_2}{\delta P} - \frac{\delta \sigma_1}{\delta P}\right)$$

Dividing by Equation 1:

$$\frac{1}{T}\frac{\delta T}{\delta P} = \frac{1}{h}\cdot\frac{\delta h}{\delta P} + \frac{K_2 - K_1}{\sigma_2 - \sigma_1}.$$

[1] Wüllner, Vol. I., p. 238.

In the present investigation δP represented a change of pressure of 1,000 pounds, $\therefore$ K_2 and K_1 represented the change in σ_2 and σ_1 for a change of 1,000 pounds in pressure.

Summary of Measurements.

1. r, the radius of the capillary tube, was measured by means of a micrometer microscope.

2. h, the difference of level of the liquid in the U-tube, was measured by means of a cathetometer. This difference of level could not be measured while the U-tube was inside the compression-chamber, because the field of view through the windows was not great enough to include the upper and lower surface at the same time. In consequence h was measured with the U-tube outside the compression-chamber.

3. a, the depth of the liquid in the U-tube below the meniscus (Fig. 5). This depth was also measured before the tube was placed in the compression-chamber.

4. $\delta h/\delta P$, the variation in height with change of pressure. The two surfaces could not be included in the field of view of the windows at one time. This was not necessary, however, for this measurement, because one arm of the U-tube being 10 mm. in diameter and the other arm of capillary dimension, a considerable change in the level of the surface in the capillary-tube would produce a very small change in the level of the surface in the larger tube. Thus the change in the height of the meniscus gave at once the value of $\delta h/\delta P$.

This method of measuring $\delta h/\delta P$, leads to the following correction.

Correction. — The liquid in the U-tube which was below the level of the meniscus contracted in volume when pressure was applied. As the contraction was the same in both arms of the U-tube, it would have introduced no error into the readings of $\delta h/\delta P$ if these readings had been made by observing the upper and lower surfaces at the same time. As, however, the reading of $\delta h/\delta P$ was made by measurements in the change of level of the meniscus only, an error was introduced. The correction was made as follows:

Let K = compressibility of the liquid per atmosphere at 25° C.

a = depth in mm. of the liquid below the meniscus (Fig. 5).

p = pressure in atmospheres.

C = correction expressed in micrometer scale-divisions (11 divisions = 1 mm.).

$C = 11$ apk.

RESULTS FOR MERCURY AND WATER.

K for mercury per atm. at 25° C. = .000004; for water = .000041.

σ_2, density of mercury at 25° C. = 13.554; σ_1, water at 25° C. = .997.

$a = 31$ mm.

$C = .09$ divisions per 1,000 pounds.

$$100\,\frac{K_2 - K_1}{\sigma_2 - \sigma_1} = .022.$$

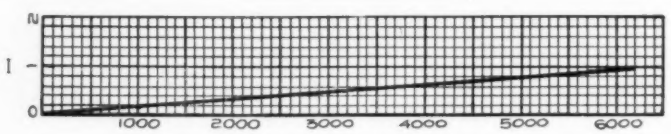
The readings are the observed changes in the height of the meniscus expressed in scale divisions. When the readings are such that they would indicate a decrease in the surface-tension, a — sign is used; otherwise, no sign is used.

Mercury and Water.

$h = 44.1$ mm.; diameters of the elliptical capillary bore 0.38 mm., 0.19 mm.

TABLE I.

Pressure in Lbs.	I.	II.	III.	IV.	V.	VI.	Mean.	Mean Corrected for Compression.	$100\,\frac{1}{h}\cdot\frac{\delta h}{\delta P}$	$100\,\frac{K_2-K_1}{\sigma_2-\sigma_1}$	$\frac{100}{T}\cdot\frac{\delta T}{\delta P}$
1,000	1.69	1.75	1.07	.73	.63	.71	1.09	1.00	.20	−.02	.18
2,000	2.88	3.14	1.83	1.64	1.18	1.02	1.95	1.77	.36	−.04	.32
3,000	4.01	4.00	2.88	2.96	1.71	1.52	2.84	2.57	.52	−.06	.46
4,000	6.17	4.96	4.58	3.91	2.15	1.96	3.95	3.59	.74	−.09	.65
5,000	7.12	6.52	6.07	3.90	2.72	2.88	4.87	4.42	.90	−.11	.79
6,000	9.58	7.33	6.55	4.42	3.19	3.24	5.73	5.19	1.06	−.13	.93



Mercury and Water.

Using a tube with a different capillary bore.
$h = 17.6$ mm.; diameter of capillary bore 0.59 mm.

TABLE II.

Pressure in Lbs.	I.	II.	III.	IV.	V.	Mean.	Mean Corrected for Compression.	$\frac{100}{h} \cdot \frac{\delta h}{\delta P}$	$100\frac{K_2-K_1}{\sigma_2-\sigma_1}$	$\frac{100}{T} \cdot \frac{\delta T}{\delta P}$
1,000 (Calc.)	.49	.30	.26	.52	.52	.42	.33	.17	−.02	.15
2,000	.72	.70	.59	1.04	1.13	.84	.66	.34	−.04	.30
3,000	1.52	.83	.83	1.69	1.69	1.31	1.04	.54	−.06	.48
4,000	1.96	1.02	.99	2.09	2.09	1.63	1.27	.66	−.09	.57
5,000	1.99		1.33	2.33	2.36	2.00	1.55	.80	−.11	.69
6,000	2.77	1.76	1.79	3.02	2.93	2.46	1.92	.99	−.13	.86

Conclusions. — (1) The surface-tension of the mercury-water surface *increases* with increase of pressure. (2) The percentage-increase is independent of the size of the capillary-tube. (3) The increase varies directly as the pressure.

Mercury and Ether.

Ether was substituted for water and the experiment repeated using the same tube as that for results in Table II. Here again it was necessary to add nitric acid to the ether, making it a three per cent. solution.

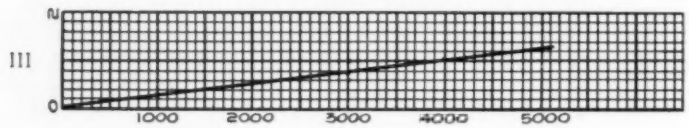
Corrections. — K for mercury = .000004, for ether = .000185; σ_2 for mercury = 13.554, σ_1 for ether = .717; $a = 31$ mm.; $C = .09$ scale-divisions per 1,000 lbs.;

$$100\frac{K_2 - K_1}{\sigma_2 - \sigma_1} = -.095.$$

TABLE III.

h = 12.75 *mm.; diameter of capillary tube* = .59 *mm.*

Pressure in Lbs.	I.	II.	III.	IV.	Mean.	Mean Corrected for Compression.	$100\frac{1}{h}\cdot\frac{\delta h}{\delta P}$	$100\frac{\bar{K}_2-\bar{K}_1}{\sigma_2-\sigma_1}$	$100\frac{1}{T}\cdot\frac{\delta T}{\delta P}$
1,000	.63	.69	.74	.65	.68	.59	.42	−.09	.33
2,000	1.08	1.26	1.46	1.16	1.24	1.06	.76	−.19	.57
3,000	1.69	1.79	1.74	1.73	1.73	1.46	1.04	−.28	.76
4,000	1.95	2.79	2.46	2.05	2.31	1.95	1.40	−.38	1.02
5,000		3.32	2.91	2.32	2.85	2.40	1.71	−.47	1.23



Conclusions. — (1) The surface-tension of the mercury-ether surface *increases* with increase of pressure. (2) The increase varies directly as the pressure.

Water-Ether.

In this case it was found undesirable to let the ether come in contact with the compression-chamber, because it rapidly dissolved the De Khotinski cement used in the joints of the pressure-gauge and about the windows. A tube of the form given in Fig. 2 was therefore used.

Ether was placed in the tube and water surrounded it. An inverted ether-column, was thus formed in the capillary tube and adjacent side-tube.

The method of observation was as above.

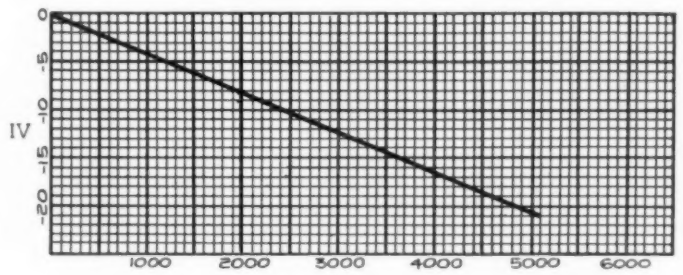
Corrections. — K for water per atm. at 25° C. = .000045; for ether = .000185; σ_1 water per atm. at 25° C. = .997; σ_2 for ether = .717; a = 8 mm.; C = 1.10 divisions per 1,000 lbs.;

$$100\,\frac{K_2-K_1}{\sigma_2-\sigma_1} = -\,3.40.$$

Results for Ether and Water: h = 13 mm.; diameter of capillary bore 0.59 mm.

TABLE IV.

Pressure in Lbs.	I.	II.	III.	IV.	Mean.	Mean Corrected for Compression.	$\frac{100}{h} \cdot \frac{\delta h}{\delta P}$	$100 \frac{K_2 - K_1}{\sigma_2 - \sigma_1}$	$\frac{100}{T} \cdot \frac{\delta T}{\delta P}$
1,000	.15	.32	.16	.20	.21	− .89	− .67	− 3.40	− 4.07
2,000	.40	.35	.32	.36	.36	−1.84	−1.39	− 6.80	− 8.19
3,000	.42	.40	.41	.38	.40	−2.90	−2.19	−10.20	−12.39
4,000	.52	.41	.44	.45	.45	−3.95	−2.99	−13.60	−16.59
5,000	.66	.56	.50	.59	.58	−4.92	−3.73	−17.00	−20.73



Graphical representation of the results given in Tables I.–V.

Conclusions. — (1) The surface-tension of the water-ether surface *decreases* with increase of pressure. (2) The decrease is proportional to the pressure.

Chloroform and Water.

Tube of form Fig. 1 used.

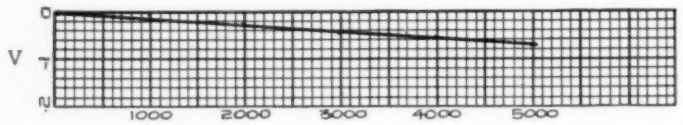
Corrections. — K for chloroform per atm. at 25° C. = .000062; for water = .000045; σ_2 for chloroform per atm. at 25° C. = 1.488; σ_1 for water = .997; a = 38 mm.; C = − 1.76 divisions per 1,000 lbs.;

$$100 \frac{K_2 - K_1}{\sigma_2 - \sigma_1} = .235.$$

Results for Chloroform and Water: h = 22.5 mm.; diameter of capillary bore = 1.04 mm.

TABLE V.

Pressure in Lbs.	I.	II.	III.	IV.	Mean.	Mean Corrected for Compression.	$100\frac{1}{h}\cdot\frac{\delta h}{\delta P}$	$100\frac{K_2-K_1}{\sigma_2-\sigma_1}$	$100\frac{1}{T}\cdot\frac{\delta T}{\delta P}$
1,000	.77	1.10	1.07	1.42	1.09	− .67	− .27	.23	−.04
2,000	1.10	1.57	1.73	2.01	1.56	−1.96	− .71	.47	−.24
3,000	1.70	2.02	3.09	2.64	2.36	−2.92	−1.18	.70	−.48
4,000	2.93	2.67	4.14	3.68	3.35	−3.69	−1.49	.94	−.55
5,000	2.54	4.07	4.86	4.92	4.09	−4.71	−1.90	1.17	−.73



Conclusion. — (1) The surface-tension of the chloroform-water surface *decreases* with the pressure. (2) The decrease is proportional to the pressure.

Carbon Bisulphide and Water.

The carbon bisulphide was Karlbaum's best redistilled three times. The tube used was of form Fig. 1.

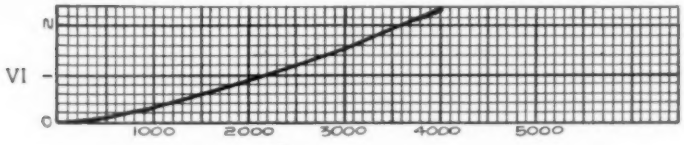
Corrections. — K for CS_2 per atm. at 25° C. = .000087, for water = .000045; σ_2 for CS_2 per atm. at 25° C. = 1.265, σ_1 for water = .997; a = 20 mm.; C = − 1.30 divisions per 1,000 lbs.;

$$100\,\frac{K_2 - K_1}{\sigma_2 - \sigma_1} = 1.36.$$

Results for Carbon Bisulphide and Water: h = 38.5. Diameter of capillary bore = 1.38 mm.

TABLE VI.

Pressure in Lbs.	I.	II.	III.	Mean.	Mean Corrected for Compression.	$100\frac{1}{h}\cdot\frac{\delta h}{\delta P}$	$100\frac{K_2-K_1}{\sigma_2-\sigma_1}$	$100\frac{1}{T}\cdot\frac{\delta T}{\delta P}$
1,000	−3.11	−4.21	−2.71	−3.34	− 4.64	−1.09	1.36	.27
2,000	−5.78	−4.81	−4.41	−5.00	− 7.60	−1.79	2.72	.93
3,000	−7.27	−6.55	−7.00	−6.94	−10.84	−2.56	4.08	1.52
4,000	−8.71	−7.50	−7.22	−7.81	−13.01	−3.07	5.44	2.37



Conclusion. — The surface tension of the carbon-bisulphide-water surface *increases* with the pressure.

Conclusions.

1. The surface-tension of the contact-surface increases with increase of pressure in the cases of mercury-water, mercury-ether, and carbon-bisulphide-water.

2. The surface-tension of the contact-surface decreases with increase of pressure in the case of ether-water and chloroform-water.

3. The percentage-change is independent of the size of the capillary tube (Tables I. and II.).

4. The percentage-change is proportional to the pressure.

In conclusion I wish to thank Professor Michelson for suggesting the problem and the method of attack, and Professor Millikan for advice during the course of the work.

Ryerson Laboratory,
The University of Chicago.

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THE ADJUSTMENT OF THE D'ARSONVAL GALVANOMETER FOR BALLISTIC WORK.[1]

BY FRANK WENNER.

DURING the past year I have used D'Arsonval galvanometers with a particular adjustment for ballistic work, and this adjustment has some very important advantages.

The coil is stripped of its dampers, so that its motion is but little damped when on open circuit. The relation between the coil constant, the suspension constant, the strength of the field, the moment of inertia of the coil and the resistance of the coil is so adjusted that the coil comes to the zero position in the least time without swinging past it. It is also desirable to have the period of the coil as large as 15 or 20 seconds. To make the period as large as this often requires the use of a small suspension or an increase in the moment of inertia of the coil. Unless the coil has a very large moment of inertia in proportion to the copper in it or swings in a very weak field the adjustment may be completed by putting the proper resistance in series with the coil. This resistance may be determined by using different values and observing the swing of the coil past the zero when released from a certain deflection. If resistances are plotted as ordinates and the swings past the zero as abscissa, the point where the curve meets the "y" axis gives the resistance.

With this adjustment the ballistic constant is e (base of Naperian log.) times as large with the coil on closed circuit as when on open circuit. The constant may be determined by the solenoid method, a condenser and standard cell or from its figure of merit and the free period of the coil. The galvanometer may be used either on closed or open circuit work with the same constant except the factor e. When used on condenser work the coil may be quickly brought to the zero by closing the coil circuit. When used on closed circuit work, such as iron testing and induction experiments, the coil returns to the zero position without attention about as fast as readings can be taken and recorded.

Other advantages which D'Arsonval galvanometers show for general work are also noticeable here.

[1] Abstract of a paper presented at the New York meeting of the Physical Society, Dec. 29–30, 1905.